



The University of Southern Queensland

## Course Specification

### Description: Advanced Mathematics

Subject	Cat-Nbr	Class	Term	Mode	Units	Campus
MAT	3100	14402	2, 2002	ONC	1.00	TWMBBA

<b>Academic Group:</b>	FOSCI
<b>Academic Org:</b>	FOS003
<b>HECS Band:</b>	2
<b>ASCED Code:</b>	010101

### STAFFING

Examiner: Chris Harman  
Moderator: Dmitry Strunin

### OTHER-REQUISITES

Pre-requisite: MAT2102 and MAT2101 and basic probability theory such as MAT 3102 or STA2301.

### RATIONALE

Some graduates will work in commercial applications of mathematics. Two areas of modern mathematics useful in finance and security are stochastic differential equations (SDEs) and cryptosystems. SDEs also apply in many other areas and have many qualitatively new characteristics. The calculus of variations is still important for its role in seeking optimal functions. These areas need the basic rigorous approach developed here.

### SYNOPSIS

This course is broadly divided into three strands. Calculus of variations introduces the finding of optimal functions and reaffirms the close connection between boundary conditions and DEs. Stochastic differential equations reflect volatility in finance and occur in other areas. The unit establishes a basic mathematical foundation for SDEs, shows some analytic solutions, and develops simple numerical schemes for simulation. Codes and cryptography are important in electronic transmission and in maintaining electronic privacy. This course applies number theory to commonly used cryptosystems.

### OBJECTIVES

On successful completion of this course students should be able to:

- apply the Euler-Lagrange equations to find optimal functions for many straightforward problems;

- derive generalisations of the Euler-Lagrange equations to analyse general optimisation problems;
- solve and interpret classes of stochastic differential equations (SDEs);
- construct and justify numerical schemes to simulate SDEs;
- apply SDEs to solve some problems in financial applications;
- explain the basis of codes and noise;
- apply public key cryptography;
- appreciate more general cryptosystems.

## TOPICS

Description	Weighting (%)
1. Euler-Lagrange equation for prescribed end conditions: challenge problems and functionals; review multi-variable extrema and Lagrange multipliers; Euler-Lagrange equation, comparison functions, fundamental lemma; special cases; straight lines minimise arclength; geodesics; soap films.	8.00
2. Extensions of basic calculus of variations: the brachistochrone has an undetermined end point; several dependent variables; minimal surfaces have several independent variables; isoperimetric problems have integral constraints; a general geodesic has non-integral constraints.	12.00
3. Financial indices appear to be stochastic processes: Brownian motion is also called a Wiener process, stochastic drift and volatility are unique, basic numerics simulate a stochastic differential equation, the binomial lattice model prices call options.	10.00
4. Ito's stochastic calculus introduced: Multiplicative noise stabilizes exponential growth, Ito's formula solves some SDEs, the Black-Scholes equation prices options.	10.00
5. Stochastic integration proves Ito's formula: the Ito integral, the Ito formula.	10.00
6. The Fokker-Plank equations describe the probability distribution: The Kolmogorov backward equation is the adjoint, solve the Black-Scholes equation stochastically.	10.00
7. Entropy & Codes: Uncertainty and entropy, Encoding, Kraft's and Macmillan's inequalities, Huffman's algorithm, noise, noisy coding theorem, linear codes.	20.00
8. Public Key Cryptosystems: One-way functions, discrete logarithms, RSA system, Rabin's system, digital signatures, breaking a cryptosystem.	20.00

### **TEXT and MATERIALS required to be PURCHASED or ACCESSED:**

Books can be ordered by fax or telephone. For costs and further details use the 'Book Search' facility at <http://bookshop.usq.edu.au> by entering the author or title of the text.

Department of Mathematics and Computing CDROM SET 1, S2, 2002 (available from the USQ Bookshop). This CD set contains course material, Windows and Linux Software for

this and various other courses. For more information about the CD sets and their use, please refer to <http://www.sci.usq.edu.au/cdrom>.

Introductory Book 2002, *Course MAT3100 Advanced Mathematics*, USQ Distance Education Centre, Toowoomba.

Selected Readings 2002, *Course MAT3100 Advanced Mathematics*, USQ Distance Education Centre, Toowoomba.

Study Book 2002, *Course MAT3100 Advanced Mathematics*, USQ Distance Education Centre, Toowoomba.

## REFERENCE MATERIALS

Reference materials are materials that, if accessed by students, may improve their knowledge and understanding of the material in the course and enrich their learning experience.

Some electronic resources for this course may be available via its home page:

<http://www.sci.usq.edu.au/courses/MAT3100>

Baxter, M. & Rennie, A 1998, *Financial Calculus. An Introduction to Derivative Pricing*, Cambridge University Press.

Bouleau, N. & Lepingue, D 1994, *Numerical Methods for Stochastic Processes*, John Wiley & Sons, New York.

Elsogotts, L 1961, *Calculus of variations*, Pergamon Press, Oxford.

Hildebrand, F.B *Methods of Applied Maths*, Dover, New York.

Kloeden, P. & Platen, E 1992, *Numerical Solution of Stochastic Differential Equations*, Springer-Verlag, New York.

Mikosch, T 1998, *Elementary stochastic calculus with finance in view*, World Scientific Publ. River Edge, New Jersey.

Weinstock, R 1974, *Calculus of variations; with applications to physics and engineering*, Dover, New York.

Welsh, D 1988, *Codes and Cryptography*, Clarendon Press, Oxford.

## STUDENT WORKLOAD REQUIREMENTS

ACTIVITY	HOURS
Assessment	30
Examinations	3
Lectures	52
Private Study	80

## ASSESSMENT DETAILS

Description	Marks Out of	Wtg(%)	Required	Due Date
ASSIGNMENT 1	7.00	7.00	Y	19 Aug 2002
ASSIGNMENT 2	7.00	7.00	Y	04 Sep 2002
ASSIGNMENT 3	7.00	7.00	Y	23 Sep 2002
ASSIGNMENT 4	7.00	7.00	Y	21 Oct 2002
3 HR RESTRICTED EXAMINATION	72.00	72.00	Y	END S2 (see note 5)

### NOTES:

5. Examination dates will be available during the Semester. Please refer to Examination timetable when published.

## OTHER REQUIREMENTS

- 1 Attendance Requirements: It is the students' responsibility to participate actively in all classes scheduled for them, and to study all material provided to them or required to be accessed by them to maximize their chance of meeting the objectives of the course and to be informed of course-related activities and administration.
- 2 Minimum Requirements to Pass the Course: To obtain a pass in this course, students must obtain at least 50% of the marks available in the examination and obtain an overall mark of at least 50% in the assignments.
- 3 Grading: Final grades for students will be determined by the addition of the marks obtained in each assessment item, weighted as in the Assessment Details.
- 4 Supplementary and Deferred Examinations: Students who obtain an overall passing mark, but who do not perform satisfactorily in the examination, may, at the discretion of the examiner, be granted a supplementary examination. Students will be granted a deferred examination only if they perform satisfactorily in the assignments. Any supplementary or deferred examinations for this course will be held during the examination period at the end of semester 3.
- 5 Assignments: The due date of an assignment is the date by which a student must despatch it to USQ. The onus is on the student to provide proof of the despatch date, if requested by the examiner. Students must retain a copy of each item submitted for assessment. This must be produced within five days if required by the examiner. In accordance with University's Assignment Extension Policy (Regulation 5.6.1), the examiner of a course may grant an extension of the due date of an assignment in extenuating circumstances. This policy may be found in the USQ Handbook, the Distance Education Student Guide and the Faculty of Sciences' Orientation Handbook for new on-campus students. All students are advised to study and follow the guidelines associated with this policy. An assignment submitted after the due date without an extension approved by the examiner, will attract a penalty of up to 20 percent of the assigned mark for each day (or part thereof) that the assignment is late. No further assignments or model solutions have been released, except in extenuating circumstances.

- 6 Examinations: Candidates should be aware that the University has policies and regulations (Regulation 5.6.2.2) about the use of unfair means and electronic devices in an examination and they should refer to them to determine whether or not actions they intend to take are acceptable to the University. Restricted Examination: Candidates will be allowed access only to specific materials in a restricted examination. The only materials that candidates may use in the restricted examination for this course (MAT3100) are: (a) writing materials (non- electronic and free from material which could give the student an unfair advantage in the examination); (b) non-programmable calculators which cannot hold textual information (students must indicate on their examination paper the make and model of any calculator(s) they use during the examination). With the approval of the examiner, candidates may take an appropriate non-electronic translation dictionary into the examination. This will be subject to perusal and may be removed from the candidate's possession until appropriate disciplinary action is completed if found to contain material that could give the candidate an unfair advantage. A list of the materials candidates may access in the restricted examination will be on the frontispiece of the examination paper.
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