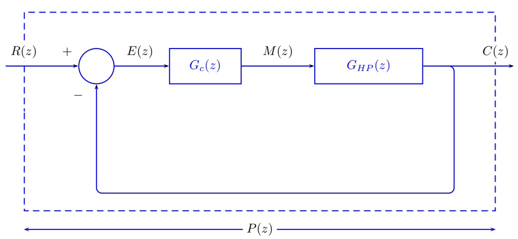
ELE3105 Computer Controlled Systems

Module 5 – Derived/Deadbeat Controllers

Derived/Deadbeat Controllers

- So far have seen PID controllers – next module is root-locus design of controllers.
- Idea of cascade controller is to take the unacceptable open-loop response (with no feedback) $G_{HP}(z)$, add feedback and a control algorithm $G_c(z)$, to give a closed-loop response $\frac{G_c(z)G_{HP}(z)}{1+G_c(z)G_{HP}(z)}$

Derived/Deadbeat Controllers



PID Controllers

- Make $G_c(z)$ a fixed transfer function (three-term)
- Optimize q_0, q_1, q_2
- It is a fixed model → can't say "I want *this* response shape" (i.e. an arbitrary response).
- Could analyze using root locus techniques.

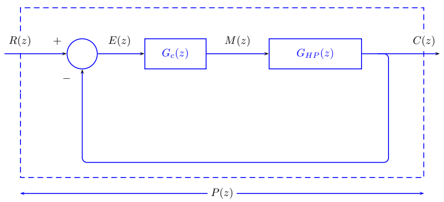
Root-Locus Controllers

- Placement of closed-loop poles at desired locations to give desired response.
- Requires knowledge of locus "behaviour" and/or locus-drawing software
- Can't have arbitrary response shapes.
- Difficult to design (analytic and iterative design process)

Derived Controllers

- Algebraic technique.
- Can design for *arbitrary* response shape (in theory...)
- In theory, this is very good.
- In practice, problems include:
  1. Overdriving the plant (saturating D/A and output electronics)
  2. Introduction of instability due to inexact modelling (can check this using root locus technque)

Derived Controllers



$$P(z) = \frac{C(z)}{R(z)} \tag{1}$$

$$E(z) = R(z) - C(z) \tag{2}$$

Derived Controllers

$$\begin{aligned} E(z) &= R(z) - P(z)R(z) \\ &= R(z)\,(1 - P(z)) \end{aligned} \tag{3}$$

$$P(z) = \frac{G_c(z)G_{HP}(z)}{1 + G_c(z)G_{HP}(z)} \tag{4}$$

$$\therefore G_c(z) = \frac{1}{G_{HP}(z)} \cdot \frac{P(z)}{1 - P(z)} \tag{5}$$

Given a desired $P(z)$, a known (fixed) $G_{HP}(z)$, can work out the required controller $G_z(z)$

Final Value Theorem

Continuous:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s \cdot F(s) \qquad (6)$$

Discrete:

$$\lim_{t\to\infty} f[n] = \lim_{z\to 1} \left(1 - z^{-1}\right) \cdot F(z) \qquad (7)$$

Final Value Theorem

Example:

$$f(t) = 1 - e^{-t} \qquad (8)$$

We know the limit as $t \to \infty = 1$

$$F(s) = \frac{1}{s(s+1)} \qquad (9)$$

Final Value Theorem

$$\text{Final Value} = \lim_{s\to 0} s \cdot \frac{1}{s(s+1)} \qquad (10)$$

$$= 1$$

$$F(z) = \frac{z\left(1 - e^{-T}\right)}{(z-1)\left(z - e^{-T}\right)} \qquad (11)$$

$$\text{Final Value} = \lim_{z\to 1} \frac{z-1}{z} \cdot F(z) \qquad (12)$$

$$= 1$$

Example 1

$$G(s) = \frac{1}{s+1} \qquad (13)$$

$$\therefore G_{HP}(z) = \left(1 - e^{-T}\right)\left(\frac{z^{-1}}{1 - e^{-T}z^{-1}}\right) \qquad (14)$$

$$E_{ss}(\text{step}) = \lim_{z\to 1} \left(1 - z^{-1}\right) E(z) \qquad (15)$$

$$= \lim_{z\to 1} \left(1 - z^{-1}\right) R(z)\left(1 - P(z)\right) \qquad (16)$$

For SSE (step) = 0, require a factor of $(1 - z^{-1})$ in $(1 - P(z))$

Example 1

Let

$$1 - P(z) = \overbrace{\left(1 - z^{-1}\right)}^{\text{For SSE=0}} \overbrace{\left(1 + a_1 z^{-1} + \cdots\right)}^{\text{polynomial 1}} \tag{17}$$

and

$$P(z) = \overbrace{\left(b_1 z^{-1} + b_2 z^{-2} + \cdots\right)}^{\text{polynomial 2}} \tag{18}$$

Two simultaneous equations to solve. Must match the powers of z when solving.

Example 1

Try

$$1 - P(z) = \left(1 - z^{-1}\right)(1) \tag{19}$$

$$P(z) = b_1 z^{-1} \tag{20}$$

Add,

$$1 = 1 - z^{-1} + b_1 z^{-1} \tag{21}$$

$$\therefore b_1 = 1 \tag{22}$$

$$\therefore P(z) = z^{-1} \tag{23}$$

The output equals the input after one sample period. Good!

Example 2

$$G(s) = \frac{1}{s+1} \tag{24}$$

$$\therefore G_{HP}(z) = \left(1 - e^{-T}\right)\left(\frac{z^{-1}}{1 - e^{-T} z^{-1}}\right) \tag{25}$$

Desired response is a first-order lag with time-constant $\tau = 0.5$ seconds.

Example 2

$$C(s) = \frac{1}{s\,(1 + 0.5s)} \tag{26}$$

$$\therefore C(z) = \left(1 - e^{-2T}\right) \frac{z^{-1}}{\left(1 - z^{-1}\right)\left(1 - e^{-2T} z^{-1}\right)} \tag{27}$$

Example 2

$$R(s) = \frac{1}{1-z^{-1}} \tag{28}$$

$$\therefore P(z) = \frac{C(z)}{R(z)} \tag{29}$$

$$= \left(1-e^{-2T}\right)\left(\frac{z^{-1}}{(1-e^{-2T}z^{-1})}\right) \tag{30}$$

Example 2

So,

$$G_c(s) = \frac{1}{G_{HP}(z)} \cdot \frac{P(z)}{1-P(z)} \tag{31}$$

$$= \left(\frac{1-e^{-2T}}{1-e^{-T}}\right)\left(\frac{z-e^{-T}}{z-1}\right) \tag{32}$$

$$= K\left(\frac{z-a}{z-b}\right) \tag{33}$$

A constant plus a pole/zero combination.

Example 3

$$G(s) = \frac{1}{s^2} \tag{34}$$

$$\therefore G_{HP}(z) = \frac{T^2}{2} \cdot \frac{z+1}{(z-1)^2} \tag{35}$$

$$\therefore G_c(z) = \frac{2}{T^2} \cdot \frac{(z-1)^2}{(z+1)} \cdot \frac{P(z)}{1-P(z)} \tag{36}$$

Example 3

$(1-P(z))$ needs a factor of $(1-z^{-1})$ for zero steady-state step error (as shown previously).

$G_{HP}(z)$ has unstable poles at $z=1$.

$\therefore (1-P(z))$ should have a factor of $(z-1)^2$ to cancel out the unstable poles and give a stable system.

Similarly for $P(z)$ and $(z+1)$

Example 3

Try

$$1 - P(z) = \left(1 - z^{-1}\right)^2 (1) \tag{37}$$

$$P(z) = \left(1 + z^{-1}\right) b_1 z^{-1} \tag{38}$$

→ End up with inconsistent solutions! ($b_1 = 2$ and $b_1 = -1$)

Example 3

Try

$$1 - P(z) = \left(1 - z^{-1}\right)^2 \left(1 + a_1 z^{-1}\right) \tag{39}$$

$$P(z) = \left(1 + z^{-1}\right) \left(b_1 z^{-1} + b_2 z^{-2}\right) \tag{40}$$

(Note matching of polynomial orders)
→ can solve this for the coefficients.

Example 4

$$G_{HP}(z) = \frac{1}{z^2 - z - 1} \tag{41}$$

Want $P(z) = z^{-1}$
Get

$$G_c(z) = \frac{1 - z^{-1} - z^{-2}}{z^{-1} - z^{-2}} \tag{42}$$

$$= \frac{M(z)}{E(z)}$$

Example 4

To get to a difference equation, need a 1 in the denominator:

$$G_c(z) = \frac{1 - z^{-1} - z^{-2}}{z^{-1} - z^{-2}} \cdot \frac{z}{z} \tag{43}$$

$$= \frac{z - 1 - z^{-1}}{1 - z^{-1}} \tag{44}$$

Gives

$$m(n) = e(n+1) - e(n) - e(n-1) + m(n-1) \tag{45}$$

But $e(n+1)$ is impossible! Future reference, called "non-causal".

Example 4

Relax requirement. Let $P(z) = z^{-2}$ Gives

$$G_c(z) = \frac{1 - z^{-1} - z^{-2}}{1 - z^{-1}} \tag{46}$$

Gives

$$m(n) = e(n) - e(n-1) - e(n-2) + m(n-2) \tag{47}$$

A solution which is "causal"