ELE3105 Computer Controlled Systems

Module 6 – Root Locus Design

Root Locus Design

- What is a Root Locus anyhow?
- Creating the Root Locus.
- Using the Root Locus to design digital controllers.

Root Locus Definition

Recall the transfer function of a closed-loop system, with forward transfer function (plant) $G(z)$ and feedback transfer function (sensor) $H(z)$ is easily shown to be:

$$\frac{K \cdot G(z)}{1 + K \cdot G(z)H(z)} \tag{1}$$

or

$$\frac{\text{direct gain}}{1 + \text{loop gain}}$$

Here,
K is the plant gain (fixed) $\times$ the controller gain (variable).
$K \cdot G(z)$ represents the plant $G_p(z)$ and the controller $G_c(z)$

Root Locus Definition

The **poles** of this system (which we now know affect the response) are given by the solution(s) of the **characteristic equation**

$$1 + K \cdot G(z)H(z) \;=\; 0 \tag{2}$$

For a fixed $G(z)$ and $H(z)$, the solutions (roots) of this equation vary with K.

Hence, the 'root locus' is just a plot of the solutions of the **characteristic equation** for K ranging from 0 to ∞.

It shows, at a glance, how the system will respond for a given controller gain K.

Root Locus Development

Given the characteristic equation

$$1 + \overbrace{\frac{K(z+0.1)}{z^2 - 0.2z + 0.1}}^{KG(z)H(z)} = 0 \qquad (3)$$

For $K = 0$, the poles are found from:

$$1 \cdot (z^2 - 0.2z + 0.1) + K(z + 0.1) = 0 \qquad (4)$$

Root Locus Development

Thus

$$z = \frac{0.2 \pm \sqrt{0.04 - 0.4)}}{2} \quad (5)$$

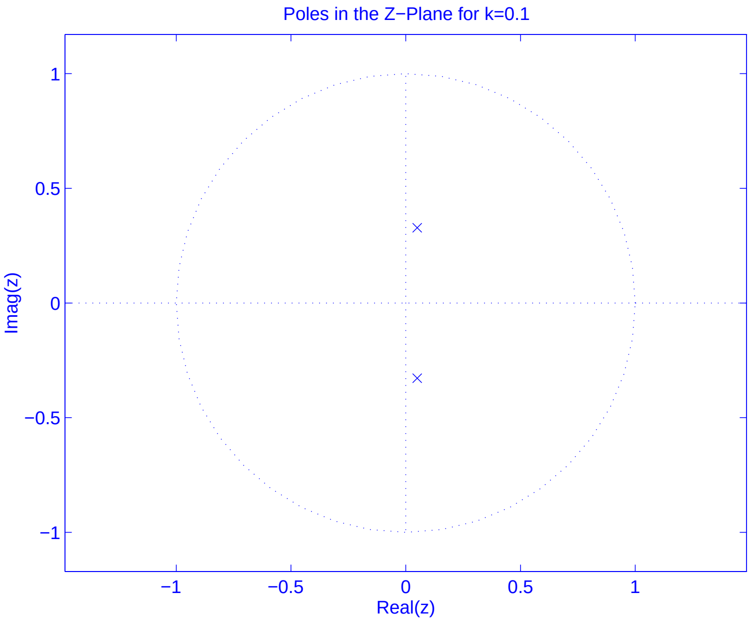
$$= 0.1 \pm \jmath 0.3 \quad (6)$$

The same as the open-loop poles.

Root Locus Development

$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

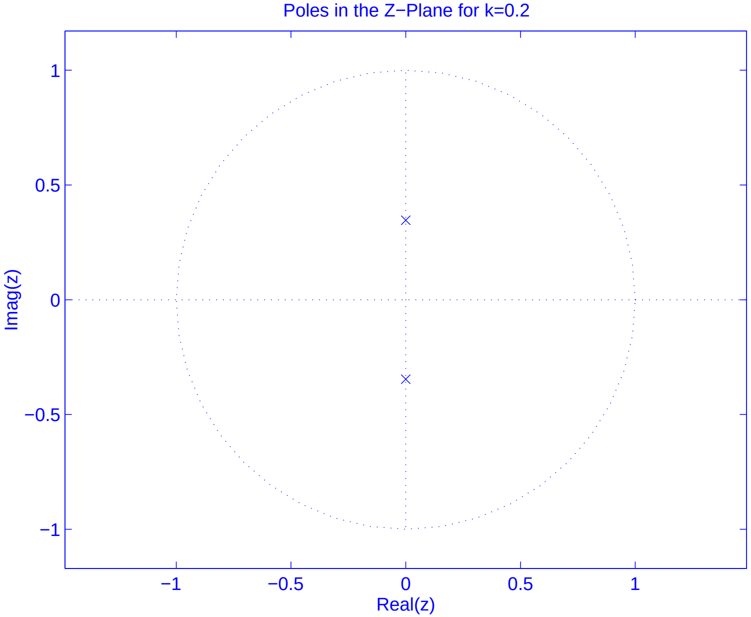
$$K = 0.1$$



Root Locus Development

$$1 + \frac{K(z + 0.1)}{z^2 - 0.2z + 0.1} = 0$$

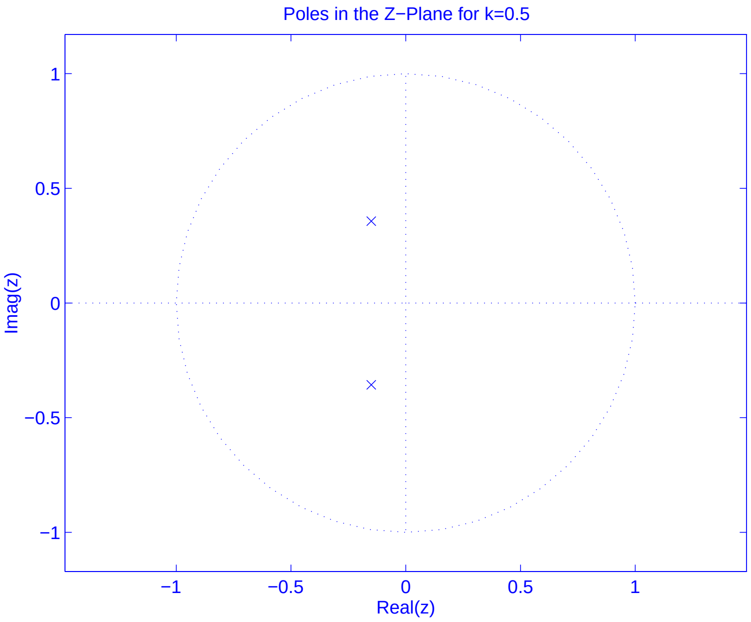
$$K = 0.2$$



Root Locus Development

$$1 + \frac{K(z + 0.1)}{z^2 - 0.2z + 0.1} = 0$$

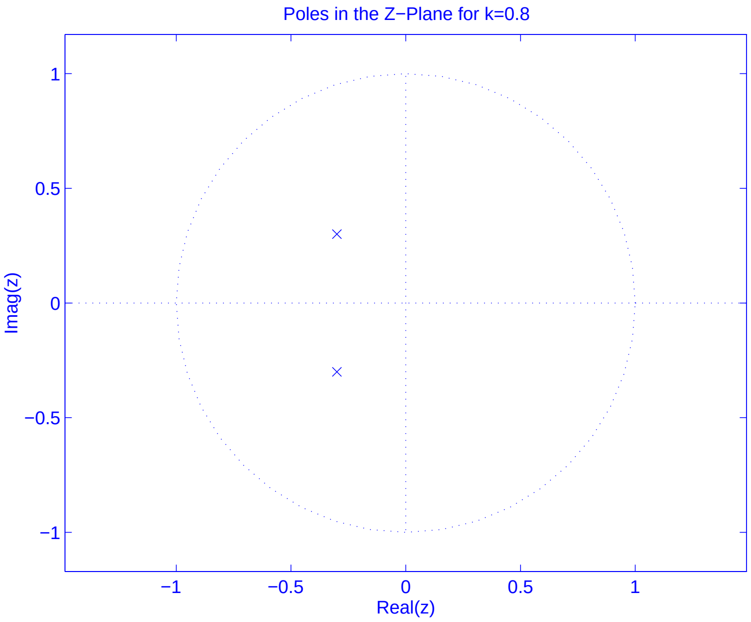
$$K = 0.5$$



Root Locus Development

$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

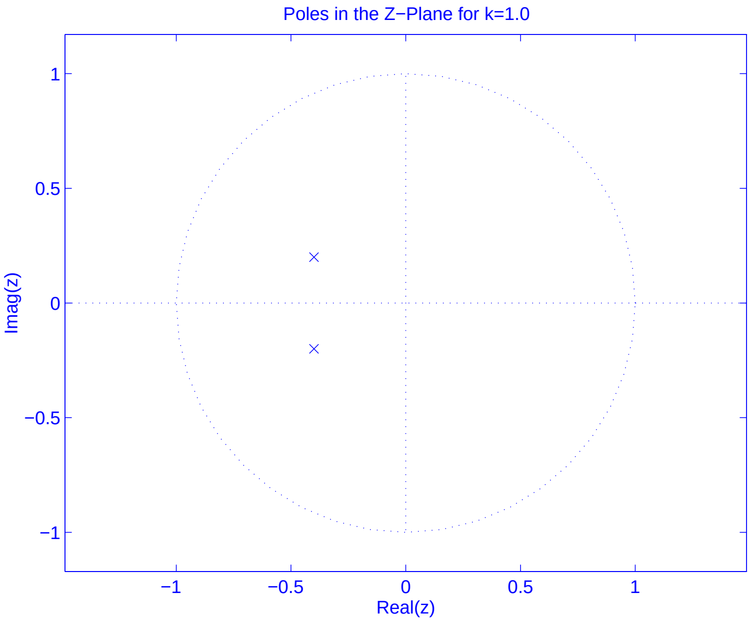
$$K = 0.8$$



Root Locus Development

$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

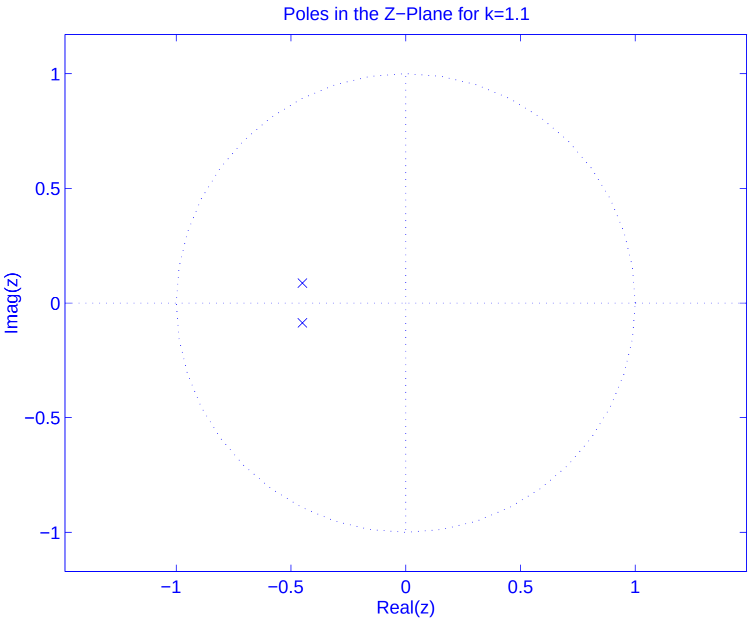
$$K = 1.0$$



Root Locus Development

$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

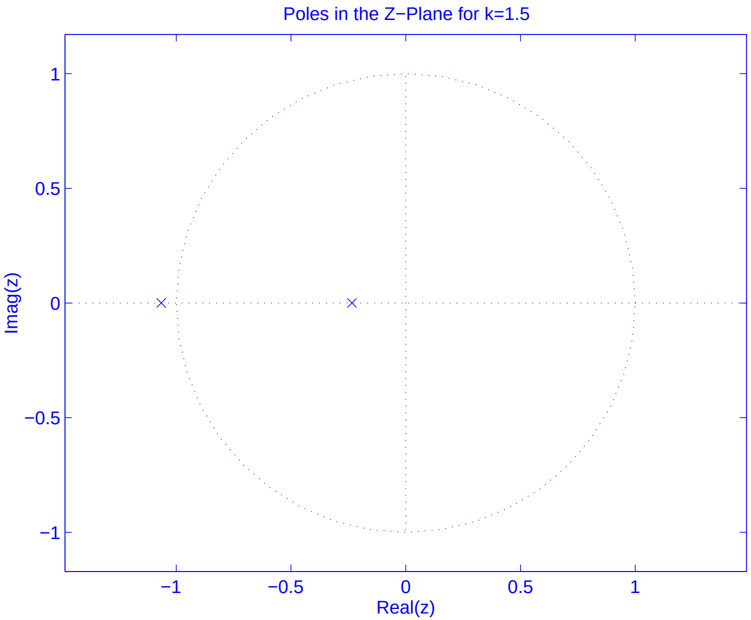
$$K = 1.1$$



Root Locus Development

$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

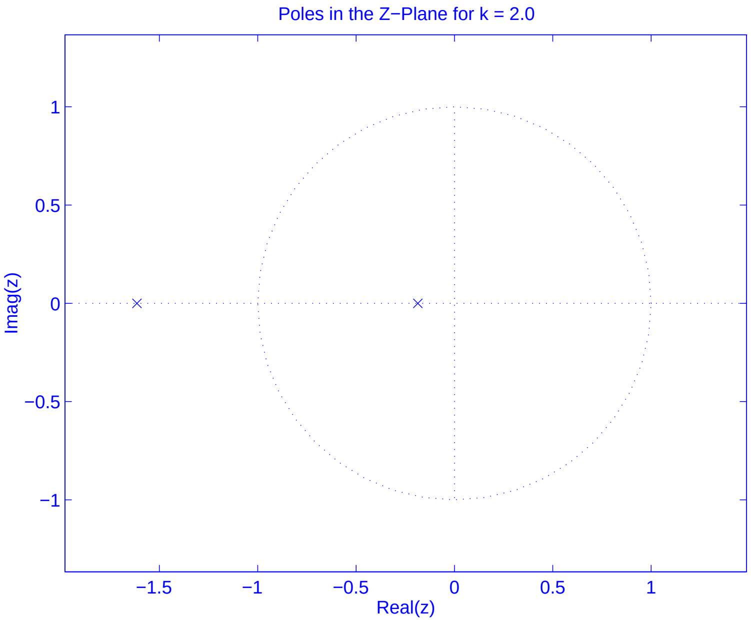
$$K = 1.5$$



Root Locus Development

$$1 + \frac{K(z + 0.1)}{z^2 - 0.2z + 0.1} = 0$$

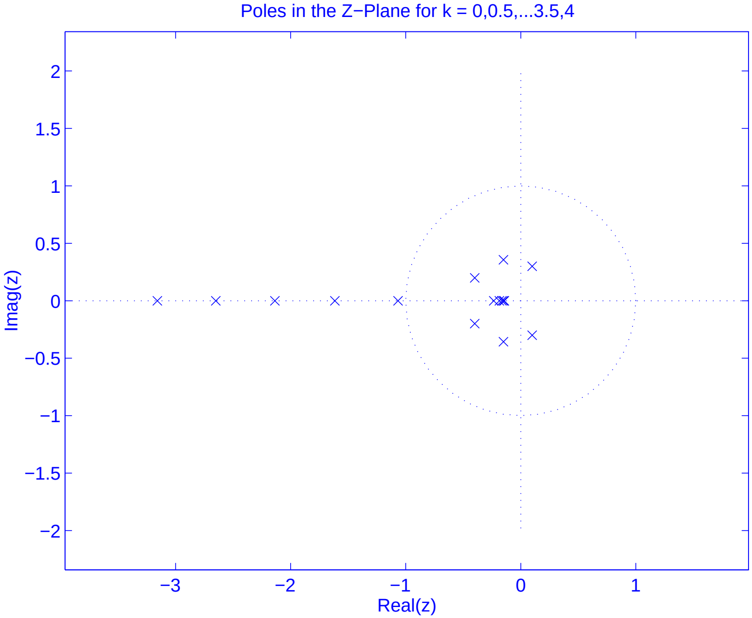
$$K = 2.0$$



Root Locus Development

$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

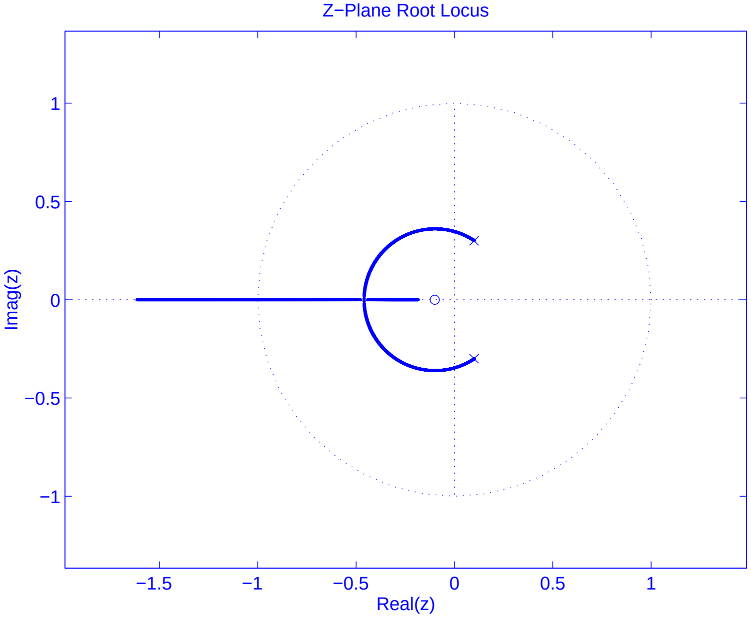
$$K = 0.0, 0.5, \cdots, 4.0$$



Root Locus Development

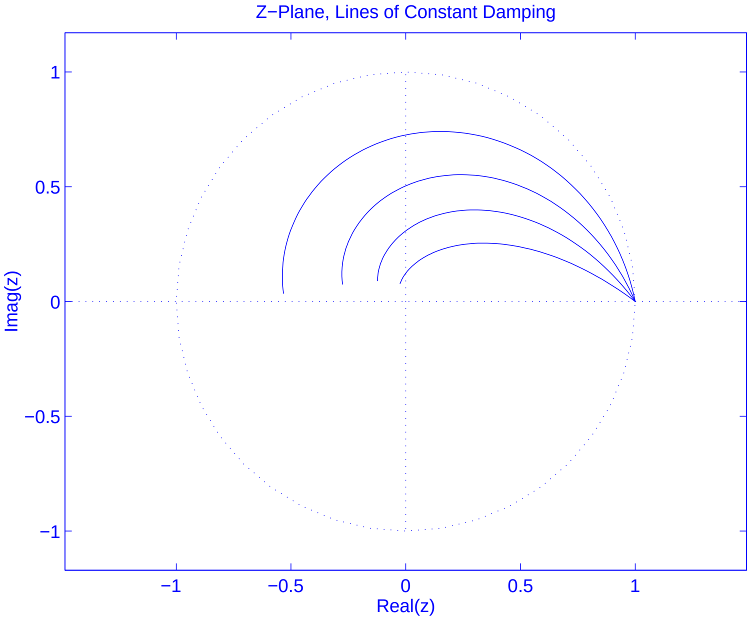
$$1 + \frac{K(z+0.1)}{z^2 - 0.2z + 0.1} = 0$$

Completed Locus



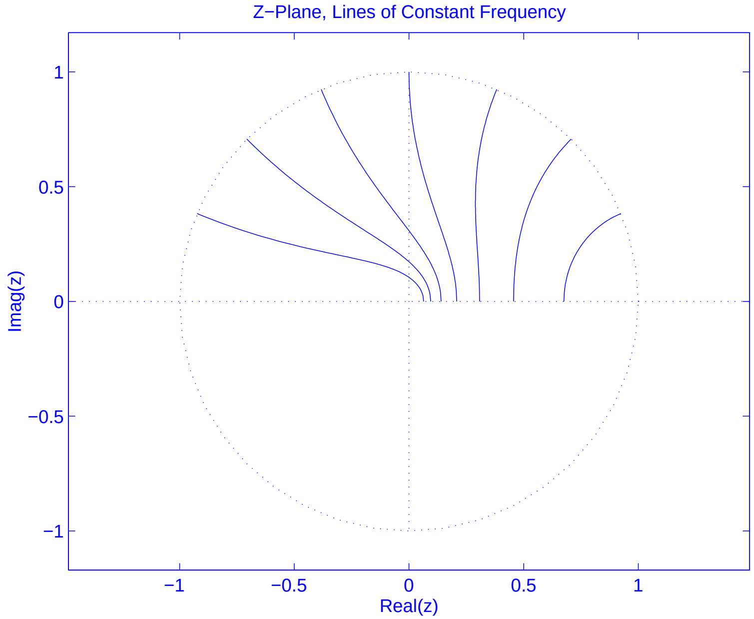
Constant Damping Ratio

$$\zeta = 0.2, 0.4, 0.6, 0.8$$



Constant Natural Frequency

$$\omega_n = 0, \frac{\pi}{8}, \cdots, \frac{7\pi}{8}$$



Root Locus Example

$$G(z)H(z) \;=\; K\frac{(z+0.1)}{(z+1.8)(z-0.3)(z-0.9)} \qquad (7)$$