University of Southern Queensland
Faculty of Engineering & Surveying

Unit Number: 70750 **Unit Name:** Dynamics II

Assessment No: 2 Internal ☑ External ☑	This Examination carries 700 of the 1000 marks total for this Unit.
Examiner: Prof T. Tran-Cong	**Moderator:**

Examination Date & Time: November 1999

Time Allowed: Perusal Ten (10) Minutes; Working Three (3) Hours

What type of exam is this? Open ☑ Closed ☐ Restricted ☐
If this is a restricted exam, what materials/equipment may students bring?

Are calculators permitted? Yes ☑ No ☐ Not Applicable ☐
If yes, what type? Programmable ☑ Non-Programmable ☐
Graph or any other special paper required? What Type
How many sheets
Are computer marked answer sheets to be used for this exam? Yes ☐ No ☑

Students are **not** permitted to retain **blue** examination papers.
Please tick on which paper this examination is to be printed Blue ☐ White ☑
Any other instructions to students and supervisors?

You may write on the examination paper during perusal time.
Attempt ALL SIX questions, which are NOT of equal value.

The following questions are from the 1999 examination paper. Here a "Guide to answer" is inserted after each question for your information.

QUESTION 1 — 120 Marks

The transformation matrix for a certain robot is given by (noting that $s_1 \equiv \sin\theta_1$, $c_{23} \equiv \cos(\theta_2+\theta_3)$, etc.)

$$ {}^0_3\mathcal{T} = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(1+c_2) \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1(1+c_2) \\ s_{23} & c_{23} & 0 & s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

A point A of the last link, (3), has the following position $[0,0,1]^T$ relative to FOR $\{3\}$. Now given that at time $t=0$ the following conditions apply:

- $\theta_1 = \theta_2 = \theta_3 = 0$;
- $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 2$ rad/s;
- $\ddot{\theta}_1 = \ddot{\theta}_2 = \ddot{\theta}_3 = 2$ rad/s/s;

Part (a) — 60 Marks
Calculate the angular velocity of (3) at time $t=0$.

(**Guide to answer:** (a) calculate the derivative ${}^0_3\dot{\mathcal{T}}$ (b) extract ${}^0_3\dot{\mathcal{R}}$ from ${}^0_3\dot{\mathcal{T}}$ (c) extract ${}^0_3\mathcal{R}$ from ${}^0_3\mathcal{T}$ (d) calculate the velocity tensor $\mathcal{V} = {}^0_3\dot{\mathcal{R}}{}^0_3\mathcal{R}^T$ (e) calculate the angular velocity vector $\vec{\Omega} = \text{vect}\,(\mathcal{V}) = [0 \quad -4 \quad 2]^T$.)

Part (b) — 60 Marks
Calculate the velocity of point A at time $t=0$.

(**Guide to answer:** Velocity of point A is simply the time derivative of its position. Thus $\vec{v}_A = \frac{d}{dt}({}^0_3\mathcal{T}\vec{P}_{A/3}) = {}^0_3\dot{\mathcal{T}}[0\;0\;1\;1]^T = [2 \quad 4 \quad 2 \quad 0]^T$.)

QUESTION 2 — 130 Marks

Part (a) — 50 Marks
Compute the modal vectors of the following 2-DOF system

$$ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} $$

(**Guide to answer:** (a) Calculate the natural frequencies by solving the eigenvalue problem $(K-\lambda M)\vec{X} = 0$) [see §14.4 of textbook, p260] $\lambda_1 = 1$ and $\lambda_2 = 3$. (b) calculate the eigenvectors $\vec{X}^{(i)}$ corresponding to the eigenvalues λ_i. $\vec{X}^{(i)}$ are called the modal vectors, which are in this case $\vec{X}^{(1)} = [1 \quad 1]^T$ and $\vec{X}^{(1)} = [1 \quad -1]^T$.)

Part (b) — 20 Marks
Sketch the mode shapes of the system.

(**Guide to answer:** A plot of the components of $\vec{X}^{(i)}$ versus the DOF is called the mode shape (i) [see the solution to 1997 exam Q9 or 1998 exam. Q2 for similar plots].)

Part (c) — 60 Marks
If the initial conditions are

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

determine the *free* (i.e. $F_1(t) = F_2(t) = 0$) response of the system.

(**Guide to answer:** It is noted that the initial conditions correspond to the second mode. Thus all DOF will vibrate with the natural frequency $\omega_2 = \sqrt{\lambda_2} = \sqrt{3}$ [see eq(14.41) of textbook, p261]. You need to use the initial conditions to determine the constants. $\vec{x}(t) = [1 \quad -1]^T \cos\sqrt{3}t$.)

QUESTION 3 — 120 Marks

Use Lagrange's method to determine the equations of motion of the multi-DOF system shown in Figure 1.

(**Guide to answer:** See §14.6.1 of textbook, p265. The mass matrix is

$$[4m \quad 0 \quad 0; 0 \quad m \quad 0; 0 \quad 0 \quad m].$$

The damping amtrix is

$$[c \quad -c \quad 0; -c \quad c \quad 0; 0 \quad 0 \quad 0].$$

The stiffness matrix is

$$[4k \quad -k \quad -k; -k \quad k \quad 0; -k \quad 0 \quad k]$$

)

QUESTION 4 — 100 Marks

Figures 2(a) and (b) describe two simple pendulums with frictionless bearing at O. Both pendulums are released from rest with a *small* initial displacement Θ as shown.

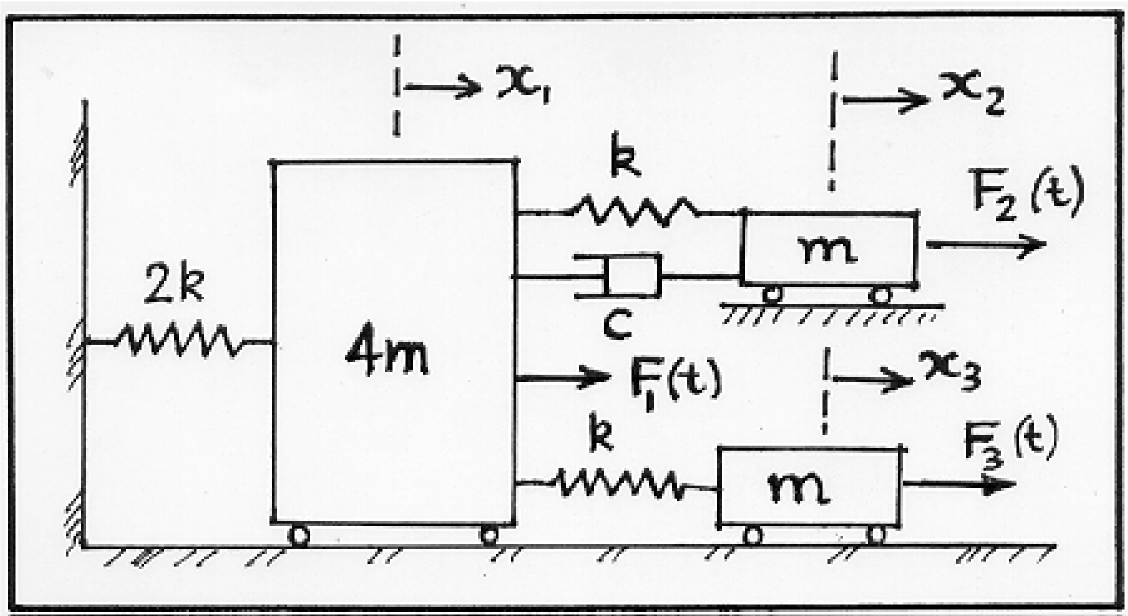

Figure 1: A 3-DOF system.

Part (a) — 50 Marks
Derive the equation of motion for each pendulum.

(**Guide to answer:** See §11.4.2 of textbook (page 202). In equation (11.23) you need to replace $\sin(\theta)$ by θ (small angle in radians assumption) and you need to calculate the inertia property I_{zz}.)

Part (b) — 50 Marks
Determine the ratio of the frequencies of the two oscillations.

(**Guide to answer:** The ratio of the two natural frequencies is $\sqrt{\frac{9}{8}}$ or $\sqrt{\frac{8}{9}}$.)

QUESTION 5 — 110 Marks

An instrument is connected to a vibrating body by a spring-damper isolator. The spring constant is $k = 40,000$ N/m and the damping constant is $c = 632.46$ Ns/m. The mass of the instrument is 10 kg. The body is vibrating at 40 Hz. Is the instrument isolated from the excitation by the vibrating body ? If so, what is the percentage isolation achieved ?

(**Guide to answer:** See StudyBook p9.4 for a similar question. You need to calculate the transmissibility T_r which is the ratio of transmitted force and the applied force. Here $T_r = 0.8256 < 1$ and hence isolation is achieved at $(1 - T_r)100\% = 17.44\%$.)

QUESTION 6 — 120 Marks

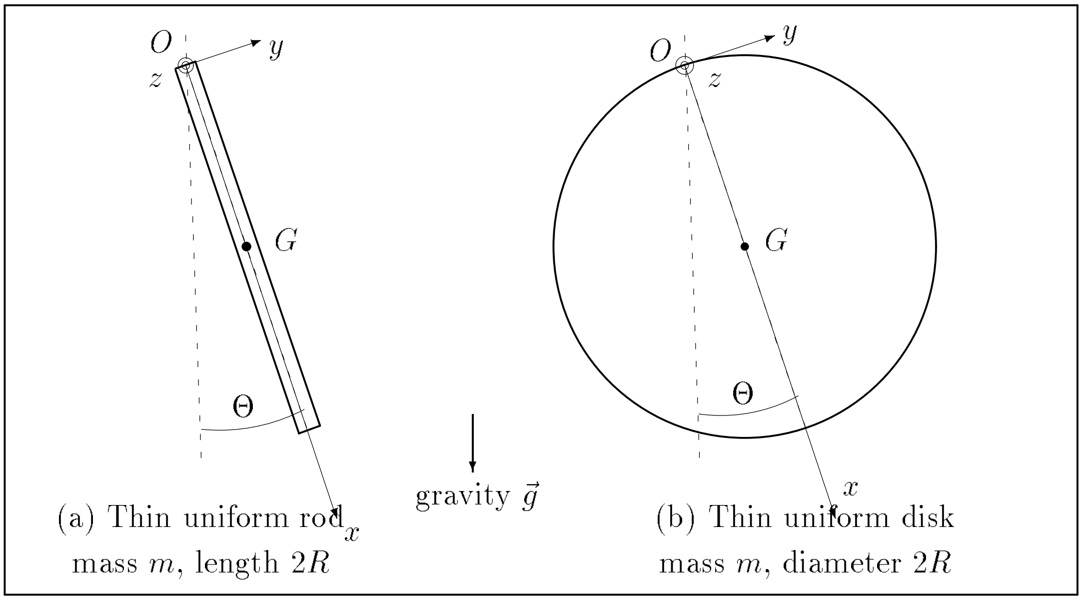

Figure 2: Two simple pendulums

A car is moving along a circular part of a road at a speed of 50 km/hr. The radius of curvature of the road is 20 m. The car is modelled as a solid parallelepiped (rectangular block) of 4 m in length, 2 m in width and 1 m in height. The mass of the car is 1000 kg.

Part (a) — 30 Marks
Determine the angular velocity of the car.

(**Guide to answer:** $\dot{\theta} = v/\rho = 0.6944$ rad/s.)

Part (b) — 40 Marks
Determine the principal inertia tensor of the car.

(**Guide to answer:** Textbook p172, formula (c) $I_{11} = 1416.7$, $I_{22} = 416.7$, $I_{33} = 1666.7$.)

Part (c) — 50 Marks
Determine the kinetic energy of the car.

(**Guide to answer:** Textbook p176, eq. (9.85) $K4 = 96,868$ J.)

End of Examination