Module A7

GENERALISING NUMBERS – ALGEBRA

Introduction

Consider the following sentences.

Gary once gave Gary's wife a gift which Gary's wife found so awful that Gary's wife threw this gift in the bin. No matter how many times Gary asked Gary's wife about the gift, Gary's wife never mentioned the gift again.

They are very difficult to read as they stand. In English we replace many of these repeated words with **pronouns** (words that replace nouns). We are very familiar and comfortable with these words in everyday life. Using pronouns, the above sentences become much easier to read.

Gary once gave **his** wife a gift which **she** found so awful that **she** threw **it** in the bin. No matter how many times Gary asked **his** wife about **it, she** never mentioned **it** again.

Whenever we see **she** in the above text, we know that this refers to Gary's wife, and whenever we see the word **it** we know it refers to the gift.

Let's look at a sentence that we have studied in module 5.

Shirley is always 3 years older than Jeffrey.

We can say that if Jeffrey is 4 years old then Shirley will be three years older than Jeffrey, meaning that Shirley is 7 years old. Or when Jeffrey is nineteen years old, Shirley will be three years older than Jeffrey, meaning that Shirley is twenty-two years old.

These sentences become hard to understand as were the sentences above. So in mathematics we introduce **pronumerals** to make the sentences easier to understand. We have already done this in module 5 where we introduced **variables.** Pronumerals and variables are the same thing. We will use the word variable throughout this module.

If we let S represent Shirley's age in years and J represent Jeffrey's age in years we know that the relationship between Jeffrey and Shirley is $S = J + 3$.

We can now rewrite the above sentences using variables (pronumerals) just as we re-wrote the original sentences using pronouns.

If $J = 4$, then $S = J + 3$, so $S = 7$. Or when $J = 19$, $S = J + 3$, so $S = 22$.

As long as we know what is meant by J and S we will understand these new sentences.

When you have successfully completed this module you should be able to:

- rearrange algebraic equations;
- solve algebraic equations involving only one variable;
- solve simultaneous equations involving two variables; and
- develop and solve equations relating to practical situations.

7.1 Simplifying expressions

This process of using letters to represent numbers is called **algebra**. You have already been using algebra throughout this unit. We used '*a*' in module 3 to represent 'any number' and when we talked about formulas in module 5 we were again using algebra. See, using algebra is really not a problem for you at all!

The word algebra comes from the name of a book *Al-jabr wa'l Muqabalah* written by an Arabic mathematician, Al-Khowarizmi, in the early ninth century. The title of the book means something like 'restoration and balancing' and this will become an important part of solving and rearranging equations in this module.

Let's look back to module 5 where we represented relationships using formulas.

Recall:
Module 5
Section 5.3

- The perimeter of a square is four times the length of one side.

  $P = 4 \times s$ where P represents the perimeter,

  $P = 4s$ and s represents the **length** of one side.

- The bank charges 8% interest on my loan.

  $I = 8\% \times A$ Where I represents the **interest charged** on the loan in dollars,

  $I = \frac{8}{100} \times A$ and A represents the **amount** of the loan in dollars.

  $I = 0.08 \times A$

- The adult weighed three times as much as the child.

  $A = 3 \times C$ where A represents the **weight** of the adult,

  $A = 3C$ and C represents the **weight** of the child.

- The grevillea was half the height of the palm tree.

  $G = \frac{1}{2} P$ where G represents the **height** of the grevillea,

  and P represents the **height** of the palm tree.

- The house was 15 metres longer than it was wide.

  $L = W + 15$ where L represents the **length** of the house,

  and W represents the **width** of the house.

Remember that the letters we have used are called variables because they can vary to take on any value that we give them.

We can also call these formulas **equations**, because the expression on the left hand side (LHS) is **equal** to the expression on the right hand side (RHS).

Let's look at some different examples.

Example

Write an equation to represent the following situation. Simplify the equation.

I think of a number and add 5. The result is 23.

Just as we define the variables when we write a formula, we must define the variables when we write an equation.

In this case we could say let N be the number I thought of.

Then, $N + 5 = 23$

Example

If Toby weighs three times as much as Harold, write an equation to represent the situation.

$T = 3 \times H$ where T represents Toby's weight,

$T = 3H$ and H represents Harold's weight.

Here are some questions for you to do.

Activity 7.1

1. Write an equation to represent each of the following situations.
    (a) A number plus 5 gives 35.
    (b) A number minus 9 is equal to 2 times the same number.
    (c) 6.5 times a number equals –3.4
    (d) Five times a number plus 7 is 22.
2. Write an equation to represent each of the following situations.
    (a) Ben is five times older than John.
    (b) The length of the rectangle was 7 metres more than the width.
    (c) Katie is five years younger than Jack.
    (d) Barry earns one third the amount of money that Harry earns.

In the previous activity you were required to write some equations. This was the same type of exercise as writing formulas that you tackled in module 5. We have also talked about expressions in past modules. Just to remind you of the difference between expressions and equations. An **expression** might involve variables, numbers and symbols (+, –, ×, ÷, $\sqrt{\ }$, ...) but **no equals sign**. An **equation** on the other hand has an **equals sign** and indicates that two expressions are equal.

Example

Write an expression to represent the following situation and then simplify the expression.

Two times a number plus five times the same number.

We must define the variable. Let the unknown number be x

Then, $2 \times x + 5 \times x$ becomes the required expression.

Recall that it is not necessary to include the multiplication sign, so we could rewrite this expression as:

$$2x + 5x$$

We call $2x$ and $5x$ **like terms** because they contain the **same power of the same variable**. Similarly, $3x^2$ and $9x^2$ are like terms because they contain the same power of the same variable. The number in front of the variable, the **coefficient,** does not influence whether terms are like or not.

When we have like terms, we can apply the distributive law to simplify the expressions.

That is, $2x + 5x$

$= 2 \times x + 5 \times x$

$= (2 + 5)x$ Applying the distributive law.

$= 7x$

Recall:
Module 2
Section 2.4.1

We could rewrite our expression as $7x$

Let's look a little more closely at like terms.

Examples

$6a$ and $2a$ are like terms.

$5x^3$ and $-7x^3$ are also like terms because they have the same power of x, that is, x^3. The coefficients, 5 and –7 do not influence our decision on like terms.

$6x^4$ and $3y^4$ are **not like terms** because they contain different variables.

$2x^2$ and $8x^3$ are **not like terms** because they have different powers of the same variable.

Example

Sort the following expressions into groups of like terms. $7x^2$, $5x$, $3x^2$, $6x$

We would group $7x^2$ and $3x^2$ together as like terms

and $5x$ and $6x$ are like terms.

Example

Simplify $3x + 2x$

Are $3x + 2x$ like terms? Yes, because they have the same power of the same variable.

Then $3x + 2x$

$= (3 + 2)x$

$= 5x$

Example

Write an expression to represent the following situation, then simplify the expression.

Three times a number squared plus nine times the same number squared.

Let the number be x.

Then, $3 \times x^2 + 9 \times x^2$

$= 3x^2 + 9x^2$

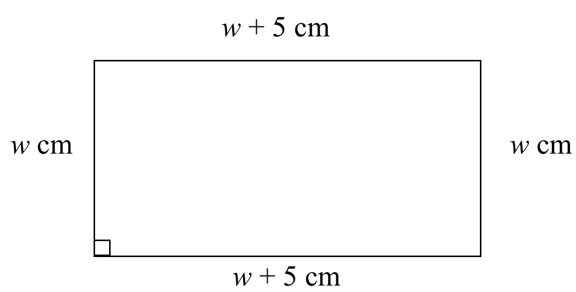
$= (3 + 9)x^2$ Applying the distributive law.

$= 12x^2$

Example

A rectangle has its length 5 cm longer than its width. If the perimeter is 62 cm, write an equation to represent the situation. Simplify the equation.

For questions like this a diagram is very helpful. Let the width of the rectangle be w cm, then the length must be $w + 5$ cm, since the length is 5 cm more than the width.



Now, the perimeter as you will recall, is the distance around an object. In this case the distance around the rectangle is 62 cm. We represent this as the equation:

$$w + 5 + w + w + 5 + w = 62$$

To simplify the expression on the left hand side (LHS) we must look for like terms. Yes, there are like terms, because the *w*'s are the same power of the same variable.

Now look at the coefficients. If there is no number in front of the variable, then the coefficient is understood to be 1. That is, if we have w it is understood that we have 1 lot of w, or $1w$.

Let's write our equation out with this new information.

$$1w + 5 + 1w + 1w + 5 + 1w = 62$$

On the LHS we can now group like terms.

$$1w + 1w + 1w + 1w + 5 + 5 = 62$$

$$(1 + 1 + 1 + 1)w + (5 + 5) = 62 \qquad \text{Using the distributive law}$$

$$4w + 10 = 62 \qquad \text{Simplifying.}$$

Remember:

- You can only add or subtract like terms.
- If a term is just x, x^2 , etc., then the coefficient is one (that is, $1x$ and $1x^2$).
- Take care when regrouping terms with negatives.

Activity 7.2

1. Sort the following expressions $3x$, $5a$, $7a^2$, $6x^2$, $7x$, $9a$, $17x$, $5x^2$, $6a^2$, $8x$, $9x^2$, $12a$, $-4x$, $-11x^2$, $-3a$, $-2x$, $2a^2$ into groups of like terms.

2. Simplify the following expressions where possible.

   (a) $5x + 6x$

   (b) $6x^2 + 11x^2$

   (c) $x + 3x$

   (d) $7x - 3x$

   (e) $9x - 14x$

   (f) $-8x - 4x$

   (g) $-3.1x + -2.4x$

   (h) $4x + 32x - 7x$

3. The length of a house is twice the width. If the perimeter of the house is 72 m write an equation to represent the situation. Simplify the equation.

4. A 75 metre long piece of rope is to be cut into two pieces. One piece is to be 15 metres longer than the other. Write an equation to represent the situation. Simplify the equation.

7.2 Rearranging equations

Let's return to Shirley and Jeffrey.

We know that Shirley is three years older than Jeffrey and that this relationship is represented by the following formula or equation.

$S = J + 3$ where S represents Shirley's age in years,

and J represents Jeffrey's age in years.

By replacing the symbol for Jeffrey's age with a numerical value we are able to find Shirley's age.

When Jeffrey was 14 years old, how old was Shirley?

That is, when $J = 14$, what does S equal? We will **substitute** 14 for the J.

$$S = 14 + 3$$

$$= 17$$

So, when Jeffrey was 14 years old, Shirley was 17 years old. (Shirley is still 3 years older than Jeffrey as she always will be.)

We call S the **subject** of this formula, since it is the variable on its own. It is also common for the subject of the formula or equation to be written on the left hand side.

What if we wanted to find Jeffrey's age for any given age for Shirley. We need a formula that has J as the subject.

Remember we said earlier that the title of the book *Al-jabr wa'l Muqabalah* meant 'restoration and balancing'. It is this principle that we will apply now.

Let's look at a simpler example to begin with.

Example

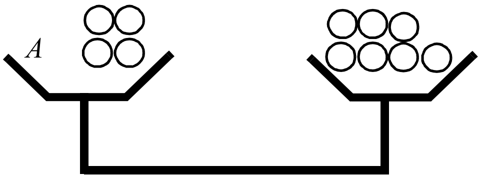
If Adam's age plus 4 equals 7, how old is Adam. (Pretend that you can't work it out!)

Representing this as an equation we get:

$A + 4 = 7$ where A represents Adam's age in years.

To find Adam's age we must make A the subject of the equation. That is, we must get the A on its own.

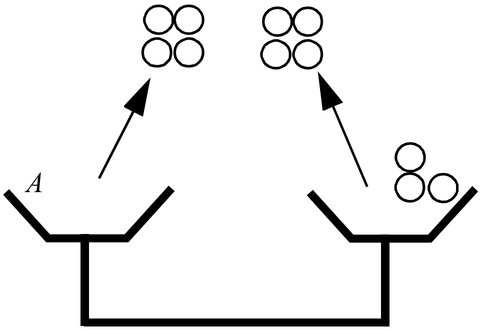
Visual Representation | Algebraic Representation



$A + 4 = 7$

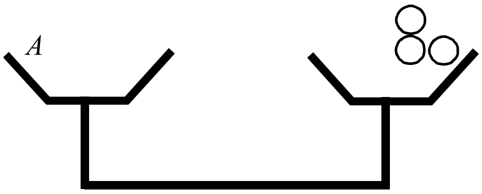
Now remembering that we must 'restore and balance', if we add or take something from one side of the scale we must do the same to the other or we will upset the balance.

If we want to leave the A on its own we must remove the 4 that is with it. If we take the 4 from one side we must do the same to the other.



$A + 4 - 4 = 7 - 4$

We are now left with:



$A = 3$

We can now say that Adam must have been 3 years old.

Let's just check that this is true.

We started by saying that:

$A + 4 = 7$ where A represents Adam's age in years.

and we **rearranged** the equation:

$A + 4 = 7$

$A + 4 - 4 = 7 - 4$ When working with equations, try to keep the equals

$A = 3$ signs under each other.

to find that $A = 3$, or we found that Adam is 3 years old.

To check this answer we can substitute our answer into the original equation. We should look at the left and right hand sides of the equation separately.

The left hand side (LHS) of the equation becomes $3 + 4$. We write this

$$\begin{aligned} \text{LHS} &= 3 + 4 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$ Therefore our answer must be correct.

So, if $A + 4 = 7$

then $A = 3$

We have rearranged the equation to make A the subject.

Let's return to our original equation.

Example

Shirley is three years older than Jeffrey.

$S = J + 3$ where S represents Shirley's age in years,

and J represents Jeffrey's age in years.

To make J the subject we must get the J on its own. To do this we must remove the 3, but we must keep the balance by removing 3 from the other side of the equation as well. Since the 3 is added we must undo the addition be subtracting 3 from both sides.

$$S = J + 3$$

$$S - 3 = J + 3 - 3$$ Keep the equals signs under each other.

$$S - 3 = J$$

$$J = S - 3$$ Writing the J on the left hand side of the equation.

Now if Shirley is 24 years old then:

$$\begin{aligned} J &= S - 3 \\ J &= 24 - 3 \\ J &= 21 \end{aligned}$$

Therefore Jeffrey is 21 years old, three years younger than Shirley as we would expect.

So far we have only looked at rearranging equations that involved addition or subtraction. Let's look at some equations involving multiplication and division.

Example

The adult weighed three times as much as the child.

$A = 3 \times C$ where A represents the **weight** of the adult,

$A = 3C$ and C represents the **weight** of the child.

Now, what if we wanted to find the weight of the child given the weight of the adult? We will need to make C the subject of the equation. This time, the C is multiplied by the 3 so to remove the 3 we must do the opposite of multiplication, that is, division.

We must divide each side by 3.

That is, $A = 3C$

$\frac{A}{3} = \frac{\cancel{3}C}{\cancel{3}}$ Cancel the 3's on the top and bottom.

$\frac{A}{3} = C$

$C = \frac{A}{3}$ Writing the subject on the LHS.

To find the age of the child we must divide the adult's age by 3.

Example

The grevillea was half the height of the palm tree.

$G = \frac{1}{2}P$ where G represents the **height** of the grevillea,

and P represents the **height** of the palm tree.

We can write this formula as:

$$G = \frac{P}{2}$$

The height of the palm tree divided by 2 gives the height of the grevillea. If we wanted to find the height of the palm tree, given the height of the grevillea we need to rearrange the equation to make P the subject.

This time the right hand side has been divided, so we need to do the opposite of this, which is multiplication. We must multiply both sides by 2 in this case.

$G \times 2 = \frac{P}{\cancel{2}} \times \cancel{2}$ Cancel the 2's on the top and bottom.

$2G = P$

$P = 2G$ Writing the subject on the LHS.

To find the height of the palm tree, we must multiply the height of the grevillea by 2.

Let's do an example that involves using two of these steps.

Example

Consider the following equation:

$$y = 3x + 5$$

To make x the subject, we need to rearrange the equation to leave x on its own.

In this example we have a multiplication ($3 \times x$) and an addition (+ 5) on the RHS.

Which do we attend to first?

Let's think about what you would do if you knew a value for x.

If you were to **substitute** this value for x into the equation, you would firstly have to multiply by 3 and then add 5 to get a value for y.

When rearranging equations we must do the opposite of these steps in the reverse order. We can think of this in terms of the following everyday occurrence.

Think of getting dressed. The first item/s of clothing you put on will be your underclothes (unless of course you are superman). This is followed by your outer clothing for the day, and finally a coat if necessary.

To undress at a later stage, instead of adding clothes you are taking them off. You also need to take off your clothes in the opposite order to that in which you put them on. That is, you remove the coat, then the outer clothes and finally your underclothes.

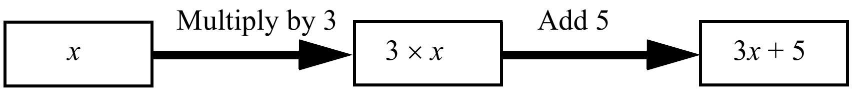
Recall the opposite of each operation.

Operation	Opposite
+	–
–	+
×	÷
÷	×

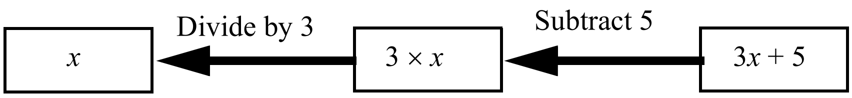
So for our example above, $y = 3x + 5$, to rearrange the equation we must subtract 5 and then divide by 3.

Let's look at this diagramatically.

If you were to substitute a value for x into the equation, you would firstly have to multiply by 3 and then add 5 to get a value for y.



To rearrange the equation we must do the opposite of the above steps in the opposite order. Reading from right to left we have:



Let's look at this algebraically,

$y = 3x + 5$ Subtract 5 from each side.

$y - 5 = 3x + 5 - 5 = 0.$

$y - 5 = 3x$ Divide **everything** on each side by 3, the y and the -5 on the LHS.

$\frac{y-5}{3} = \frac{3x}{3}$ Cancel the 3's on the RHS.

$\frac{y-5}{3} = x$

$x = \frac{y-5}{3}$ Writing the subject on the LHS.

Example

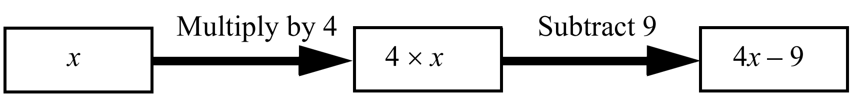
Make x the subject of the following equation.

$$y = 4x - 9$$

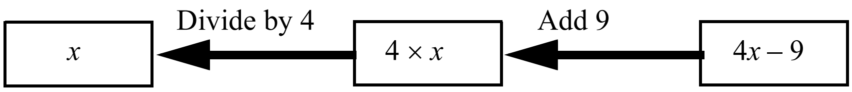
Firstly we must think about what steps we would take if we knew a value for x and were trying to find a value for y.

In words, we have to multiply by 4 and then subtract 9. So to rearrange we would add 9 and then divide by 4.

Diagramatically,



To rearrange the equation we must do the opposite of the above steps in the opposite order. Reading from right to left we have:



And algebraically,

$$\begin{aligned} y &= 4x - 9 && \text{Add 9 to both sides.} \\ y + 9 &= 4x - 9 + 9 && -9 + 9 = 0 \\ y + 9 &= 4x && \text{Divide } \textbf{everything} \text{ on each side by 4.} \\ \frac{y+9}{4} &= \frac{\not{4}x}{\not{4}} && \text{Cancel the 4's on the RHS.} \\ x &= \frac{y+9}{4} && \text{Writing the subject on the LHS.} \end{aligned}$$

Example

Make x the subject of the following equation.

$$y = 8 - 4x$$

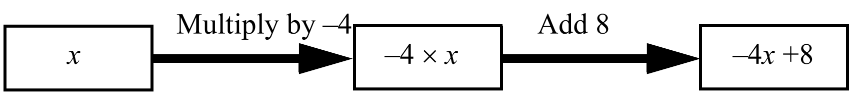
You should note that when looking at equations or expressions, the sign **in front** of a number is the one attached to the number.

In the above example there is no sign in front of the 8 so it understood that this is positive 8. The sign in front of the $4x$ is negative so this is $-4x$.

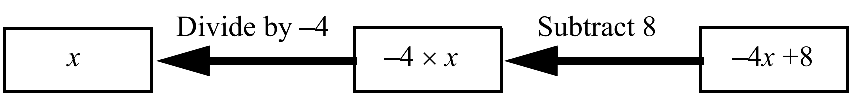
Firstly we must think about what steps we would take if we knew a value for x and were trying to find a value for y.

In words, we have to multiply by –4 and then add 8. So to rearrange we would subtract 8 and then divide by –4.

Diagramatically,

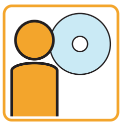


To rearrange the equation we must do the opposite of the above steps in the opposite order. Reading from right to left we have:



And algebraically,

$y = 8 - 4x$	Subtract 8 from both sides.
$y - 8 = 8 - 4x - 8$	$8 - 8 = 0$
$y - 8 = -4x$	Divide **everything** on each side by –4.
$\dfrac{y-8}{-4} = \dfrac{\cancel{-4}x}{\cancel{-4}}$	Cancel the –4's on the RHS.
$x = \dfrac{y-8}{-4}$	Writing the subject on the LHS.



Check out the resource CD. It has some extra materials to help make sense of this algebra. Commence by doing the 'Getting to the answer' program, this will show you how to use backtracking to solve equations. Then view the 'Rearranging formula 1' and 'Rearranging formula 2' presentations, which both give more explanation on how to rearrange formulas.

Activity 7.3

1. Rearrange the following equations to make the variable in the bracket the subject.

   (a) $R = 4a$ $\quad(a)$

   (b) $x + 2 = y$ $\quad(x)$

   (c) $\dfrac{5}{b} = k$ $\quad(b)$

   (d) $y = 5x + 2$ $\quad(x)$

   (e) $7x - 8 = y$ $\quad(x)$

   (f) $\dfrac{6}{t} + 7 = s$ $\quad(t)$

   (g) $\dfrac{x+8}{3} = y$ $\quad(x)$

2. The perimeter of a square is four times the length of one side.

   $P = 4 \times s$ where P represents the perimeter,

   $P = 4s$ and s represents the **length** of one side.

   Rewrite this formula to make s the subject.

3. Before the invention of mechanical clocks, candles were sometimes used to measure the passage of time. A formula for the height of such a candle related to time is given below.

   $h = 10 - 2t$ where h equals the height of the candle in centimetres,

   and t equals the time in hours that the candle has been burning.

   Rewrite this equation to make t the subject.

4. Economists have come up with the following formula relating the consumption of milk to the number of children in the family.

   $m = 3.5 + 2.5x$ where m is the number of litres of milk consumed per week,

   and x is the number of children in the family.

   Rewrite this formula to make x the subject.

5. We know that the circumference of a circle is equal to π times the diameter.

   $C = \pi d$ where C represents the circumference of the circle,

   and d represents the diameter of the circle.

   Express this formula with d as the subject.

Now that you are familiar with rearranging some types of equations, let's look at solving equations.

7.3 Solving equations

We have already looked at skills involved in rearranging equation to give a different subject. We will use these same skills to solve equations.

Example

In the previous activity we looked at a formula relating the consumption of milk to the number of children in the family.

$m = 3.5 + 2.5x$ where m is the number of litres of milk consumed per week,

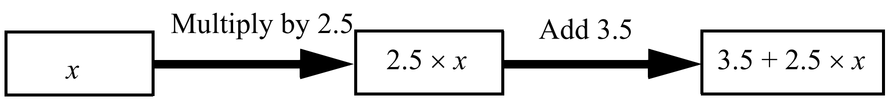
and x is the number of children in the family.

The milkman knows that the Cowper family buys 11 litres of milk each week. He is thus able to work out the number of children in the Cowper family.

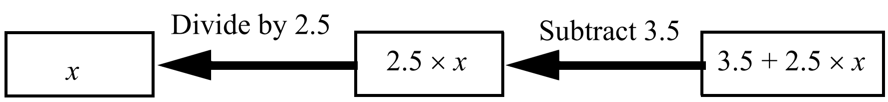
The number of litres used in one week is 11, so this gives us $m = 11$ which we substitute into the equation.

$$11 = 3.5 + 2.5x$$

Using our reasoning from before, if we knew a value for x we would follow these steps:



To rearrange the equation we must do the opposite of the above steps in the opposite order.



Remember also that we must keep the balance. Whatever we do to one side, we must do to the other.

$$11 = 3.5 + 2.5x$$

Remember to keep the equals signs under each other.

Subtract 3.5 from both sides.

$$11 - 3.5 = 3.5 + 2.5x - 3.5$$

$$7.5 = 2.5x$$

Divide both sides by 2.5

$$\frac{7.5}{2.5} = \frac{2.5x}{2.5}$$

$$3 = x$$

Write the variable on the left.

$$x = 3$$

The great thing about solving equations is that you can check that your answer is correct.

We think the answer to the above question is $x = 3$ so we should check that this satisfies the above equation.

To do this we look at the left hand side (LHS) and the right hand side (RHS) of the equation separately.

Now, $11 = 3.5 + 2.5x$

When $x = 3$, LHS $= 11$

$$\begin{aligned} \text{RHS} &= 3.5 + 2.5x \\ &= 3.5 + 2.5 \times 3 \\ &= 3.5 + 7.5 \\ &= 11 \\ &= \text{LHS} \end{aligned}$$

Since the LHS is equal to the RHS the answer must be correct.

So if the milkman delivers 11 litres of milk, he can assume there are 3 children in the household.

Look carefully at the format of the above solution. It is in correct and logical form. Just like in English you are encouraged to write correct and logical sentences. Be sure to practice the correct format in mathematics.

Example

Solve $x - 2 = -7$

$$x - 2 = -7$$

Add 2 to both sides.

$$x - 2 + 2 = -7 + 2$$

$$x = -5$$

Check: LHS $= x - 2$

$= -5 - 2$

$= -7$

$=$ RHS

Example

Solve $2x + 8 + 3x = 3$

$$\begin{aligned} 2x + 8 + 3x &= 3 \\ 2x + 3x + 8 &= 3 \\ (2+3)x + 8 &= 3 \\ 5x + 8 &= 3 \\ 5x + 8 - 8 &= 3 - 8 \\ 5x &= -5 \\ \frac{\not{5}x}{\not{5}} &= \frac{-5}{5} \\ x &= -1 \end{aligned}$$

Group together like terms.

Subtract 8 from each side.

Divide both sides by 5

Check: LHS $= 2x + 8 + 3x$

$= 2 \times -1 + 8 + 3 \times -1$ Must do multiplication before addition.

$= -2 + 8 - 3$

$= 3$

$=$ RHS

Example

Solve $3(x + 1) = 12$

If you know a value for x, you would add 1 and then multiply by 3. So we do the opposite of these in the opposite order. That is, divide by 3 and then subtract 1.

$$\begin{aligned} 3(x+1) &= 12 \\ \frac{\not{3}(x+1)}{\not{3}} &= \frac{12}{3} \\ x + 1 &= 4 \\ x + 1 - 1 &= 4 - 1 \\ x &= 3 \end{aligned}$$

Divide everything on each side by 3

Take 1 from each side.

Check: LHS $= 3(x + 1)$

$= 3 \times (3 + 1)$

$= 3 \times 4$

$= 12$

$=$ RHS

Example

Solve $3x + 2 = 5x + 4$

In this question, the first step will be to get all the terms involving x together on one side.

$3x + 2 = 5x + 4$ Remove the $5x$ from the RHS.

$3x + 2 - 5x = 5x + 4 - 5x$ Group the like terms.

$3x - 5x + 2 = 4$

$(3 - 5)x + 2 = 4$

$-2x + 2 = 4$ Subtract 2 from each side.

$-2x + 2 - 2 = 4 - 2$

$-2x = 2$ Divide both sides by -2

$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{2}{-2}$

$x = -1$

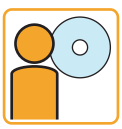
Check: LHS $= 3x + 2$ RHS $= 5x + 4$

$= 3 \times -1 + 2$ $= 5 \times -1 + 4$

$= -3 + 2$ $= -5 + 4$

$= -1$ $= -1$

Since RHS is equal to LHS the solution is correct.



Check out the resource CD and in particular the 'Getting to the answer section', which provides more explanation on how to solve equations.

Activity 7.4

1. Solve the following equations. Verify your answer each time by substituting it into the original equation.

   (a) $x + 3 = 9$

   (b) $x - 2 = 5$

   (c) $5x = 15$

   (d) $\frac{x}{2} = 6$

   (e) $-3 + x = 7$

   (f) $2.4 - x = 3.6$

   (g) $2x + 1 = -8$

   (h) $\frac{x}{5} - 2 = 7$

   (i) $5 + 2x = 1$

   (j) $10 - 3x = 1$

   (k) $\frac{6}{t} + 2 = 12$

   (l) $2.5x + 3.5 = 7$

   (m) $4(3x - \frac{4}{9}) = 3$

   (n) $2x + 3 + 5x = 17$

   (o) $5x + 3 - 2x = 21$

   (p) $2x + 1 = 3x - 5$

   (q) $0.5x - 4.1 = 3.9 - 0.7x$

2. You are now in a position to solve this equation you formed in Activity 7.2. The length of a house is twice the width. If the perimeter of the house is 72 m write an equation to represent the situation. Solve the equation to find the length of the house.

3. A 63 metre long piece of rope is to be cut into two pieces. One piece is to be 17 metres longer than the other. Write an equation to represent the situation. Solve the equation to find the length of the two pieces of rope.

4. A rectangle is 4 cm longer than it is wide. If the perimeter is 52 cm, write an equation to represent this situation. Solve the equation to find the length of the rectangle.

5. I think of a number, double it and add 7. The result is 59. What number did I think of?

6. One third of a number added to 23, gives a total of 34. What is the number?

7. If I double Rebecca's age, subtract three and divide the result by five, the answer is 5. How old is Rebecca?

8. Samantha went to the Exhibition with $50 from her mother, which was to buy one sample bag for each of her brothers.

   (a) After buying x sample bags at $7.75 each, Samantha has P dollars left. Write a formula connecting P and x.

   (b) How many brothers did Samantha buy bags for if she has $19 left?

9. Mrs Manykids went to buy sweets and chips for her youngest's birthday party. The packets of sweets cost 2.5 times as much as the packets of chips, and Mrs Manykids spent $42 in buying equal numbers of each item. If the chips cost $1.20 a packet, and each child received one packet of sweets and one packet of chips, how many children did Mrs Manykids buy for?

10. Solve the following equations. Verify your answer each time by substituting it into the original equation.

   (a) $\frac{2}{5} + \frac{7}{3}t = 5$

   (b) $\frac{2}{3}x + 7 = \frac{1}{2}x - 3$

   (c) $\frac{7}{8} - \frac{3}{8}x + 4 = \frac{3}{4} + 2x$

7.4 Equations involving powers and roots

So far we have not looked at equations that involved powers and roots. We will not look at these in great detail in this unit but it is worthwhile having a brief look at this type of equation.

Recall the following example from module 5.

Example

The formula for the area of a square is

$A = s^2$ where A represents the area of the square,

and s represents the side length of the square.

This formula allows us to find the area of a square of any given side length. But what if we are given an area and we need to know the side length?

We need to rearrange the formula $A = s^2$ to make s the subject.

So far the equations we have rearranged have only involved the four basic operations, addition, subtraction, multiplication and division. We now have an equation that involves a squared term. In module 2 we looked at the idea that squares and square roots are the opposites of each other.

Recall:
Module 2
Section 2.2.3

So to remove the square from the s

we need to take the square root of each side.

$$\begin{aligned} A &= s^2 \\ \sqrt{\mathrm{A}} &= \sqrt{s^2} \\ \sqrt{\mathrm{A}} &= s \\ s &= \sqrt{\mathrm{A}} \end{aligned}$$

Let's look at this type of question a little more closely.

Example

Solve for x, $x^2 = 4$

Let's think about this question for a while.

Can you think of a number that you could square and get the answer 4? ....

Think a little harder and see if you can come up with another number ....

You should have thought of the numbers 2 and –2. There are actually two solutions to this question and we need to reflect this in our answers to questions where we take square roots.

$x^2 = 4$ To get x on its own we need to take the square root of each side.

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$ We place the symbol ± in front of the square root sign to indicate that there are two possible answers, one positive and one negative.

Check:	LHS $= x^2$		LHS	$= x^2$
when $x = 2$	$= 2^2$		when $x = -2$	$= (-2)^2$
	$= 4$			$= 4$
	$=$ RHS			$=$ RHS

Example

Make n the subject of this formula

$K = (4n+1)^2$ Take the square root of both sides.

$\pm\sqrt{K} = 4n + 1$ Both the positive and negative root must be given.

$\pm\sqrt{K} - 1 = 4n + 1 - 1$

$\pm\sqrt{K} - 1 = 4n$

$\frac{\pm\sqrt{K} - 1}{4} = \frac{\cancel{4}n}{\cancel{4}}$ Divide **everything** on each side by 4.

$\frac{\pm\sqrt{K} - 1}{4} = n$

$n = \frac{\pm\sqrt{K} - 1}{4}$

You could also see this written as $n = \frac{-1 \pm \sqrt{K}}{4}$

The two terms on the top of the fraction have been placed in the opposite order. It is quite acceptable to rearrange terms like this as long as you remember to keep the sign that is in front of a term with the term. In this example the negative sign belongs to the 1 and should accompany it wherever it goes. Similarly the ± belongs to the $\sqrt{K}$ and moves with it.

Example

Make n the subject of this formula.

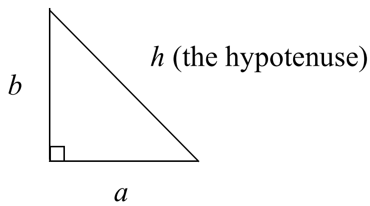
$$
\begin{aligned}
y &= \sqrt{4n+3} && \text{Square both sides remembering to square all of the RHS.}\\
y^2 &= 4n+3\\
y^2-3 &= 4n+3-3\\
y^2-3 &= 4n\\
\frac{y^2-3}{4} &= \frac{\cancel{4}n}{\cancel{4}}\\
\frac{y^2-3}{4} &= n\\
n &= \frac{y^2-3}{4}
\end{aligned}
$$

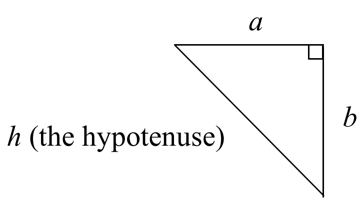
7.4.1 Pythagoras' Theorem

A formula that relies on your knowledge of manipulating equations with powers and roots is **Pythagoras' Theorem.** Pythagoras was a Greek mathematician who lived about 540 B.C. He was one of the first true mathematicians who saw mathematics in a theoretical sense rather than just a practical means of calculating taxes and interest or areas of fields. He travelled widely and established the mystical Pythagorean society which prohibited the eating of beans (one can only speculate as to why this might have been so) and whose motto was *All in number.*

Pythagoras' Theorem states that the length of the hypotenuse squared equals the sum of the squares on the other two sides. Recall from a previous module that the hypotenuse is the side opposite the right angle. It is always the longest side on the triangle.

Recall:
Module 4
Section 4.2.2





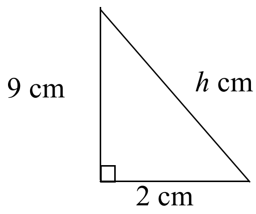
If we represent the other two sides by a and b we could write Pythagoras' theorem in symbols.

$$h^2 = a^2 + b^2$$

To be able to use this formula, all the lengths of the sides must be measured in the same units.

Example

What is the length of the hypotenuse of the following triangle?



From Pythagoras' Theorem

$$h^2 = a^2 + b^2$$

$$a = 2,\ b = 9,\ h = ?$$

It wouldn't matter which side we let be a and which side we let be b.

So, $h^2 = 2^2 + 9^2$

$h^2 = 4 + 81$

$h^2 = 85$

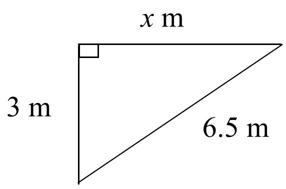
So, $h = \sqrt{85} \approx 9.22$ Note that the negative square root in this case has no meaning so we disregard it

The length of the hypotenuse is approximately 9.22 cm

It is also possible to use this rule to find the lengths of sides other than the hypotenuse.

Example

Find the length of the unknown side



Pythagoras' Theorem states

$$h^2 = a^2 + b^2$$

$$a = 3,\ b = x,\ h = 6.5$$

So, $(6.5)^2 = 3^2 + x^2$

$42.25 = 9 + x^2$ Take 9 from both sides.

$42.25 - 9 = x^2$

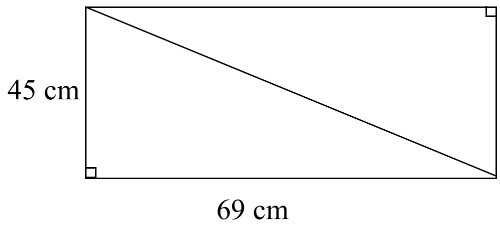
$x^2 = 33.25$

So, $x = \sqrt{33.25} \approx 5.8$ Again we disregard the negative square root.

So the length of the unknown side is approximately 5.8 m

Example

A carpenter is making the top of a table from a piece of timber. The length of the piece of timber is 69 cm and the width is 45 cm. How can the carpenter check to see if it is a rectangle (that he has made the angles 90°)?



The carpenter should measure the diagonal length of the timber.

Let the length of the diagonal be x cm

Using Pythagoras' theorem

$$x^2 = 45^2 + 69^2$$

$$x^2 = 2\,025 + 4\,761$$

$$x^2 = 6\,786$$

$$x = \sqrt{6\,786}$$

$$x \approx 82.4$$

If the diagonal is approximately 82.4 cm long then the table top will be rectangular.

Activity 7.5

1. Make a the subject of the following equations.

   (a) $b = 2a^2 + 1$

   (b) $c = 2(3a - 7)^2$

   (c) $T = \sqrt{1.5 - 2.6a}$

   (d) $m = \dfrac{\sqrt{a^2 + 2}}{5}$

   (e) $B = 6\sqrt{3a + 7}$

2. If I stand on top of a cliff S metres high and drop a stone, it will take t seconds to reach the ground. The height of the cliff can be found using the following formula.

   $$S = 4.9\,t^2$$

   (a) Rearrange this formula to make t the subject.

   (b) Use this new formula to find the time to reach the ground for a cliff of height 650 metres. Round your answer to the nearest tenth of a second.

3. Observation towers are often built in forests to allow bushfires to be spotted quickly. The distance (D) in kilometres that can be seen from a tower of height h metres is given by:

   $$D = 8\sqrt{\frac{h}{5}}$$

   (a) Rearrange this formula to make h the subject.

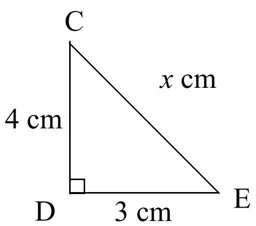
   (b) Find the height of the tower to the nearest metre if a distance of 15 kilometres can be seen from the tower.

4. Form an equation for the following relationship and solve the equation.

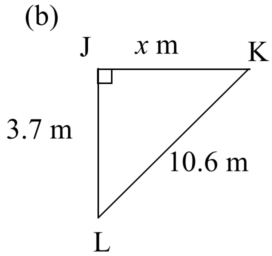
   I think of a number, square it and multiply the result by 4. The answer is 36. What was the number I thought of?

5. For the following right angled triangles, find the unknown length (to two decimal places if necessary).

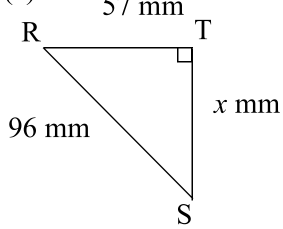
(a)



(b)



(c)



6. A fence builder needs to brace a 2 m by 3 m rectangular gate with a piece of timber placed diagonally. How long should the timber be cut (to the nearest mm)?

7. A 15 metre wire is used to support a flag pole. If the wire is attached to the ground 9 m from the base of the flag pole, how high up the pole is the wire attached?

8. Each day Joe walks home past a rectangular sporting field. If the field is being used, Joe walks along two sides of the field. If the field is not being used he cuts across from corner to corner. If he takes 300 steps along one edge of the field and 500 steps along the other edge, approximately how many steps does Joe save by cutting across the field?

9. A ten metre ladder is resting against a house, with the foot of the ladder 2.4 metres from the house. If the foot of the ladder is pulled 2.1 more metres from the house, how far down the side of the house will the ladder move? (Answer to the nearest tenth of a metre)

10. Make p the subject of the following equations.

(a) $l = \frac{2}{3}p^2 + 4$

(b) $l = \dfrac{6\sqrt{3p - \frac{3}{4}}}{4}$

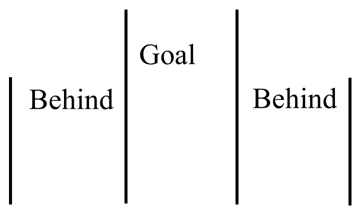
(c) $l = -6\left(\frac{5}{3}p^2 + 8\right)$

7.5 Simultaneous equations

Have you ever looked at the scores in Australian Football and wondered what they all meant? Of course, if you are into AFL then you will be very familiar with these sorts of scores. For the purposes of this exercise you will need to pretend this is all new to you.

Brisbane Lions:	17	1	103
Collingwood:	9	14	68

I did work out that there were two ways to score in Australian Rules: a goal and a behind.



Now I also worked out when listening to the TV reporter announce the results that the first score represented the goals, the second score represented the behinds and the final score represented the total score for the team. Since these two numbers didn't add up to give the total it set me to wondering what score a goal was worth and what score a behind was worth.

Brisbane Lions: $17 \times$ the score for goals + $1 \times$ the score for behinds = 103

Collingwood: $9 \times$ the score for goals + $14 \times$ the score for behinds = 68

Let the score for goals be represented by G and the score for behinds be represented by B.

I can now rewrite the equations for each team's scores.

Brisbane Lions: $17 \times G + 1 \times B = 103$

Collingwood: $9 \times G + 14 \times B = 68$

Omitting the multiplication signs we have:

$17G + 1B = 103$ (1) It is normal to give each of the equations a number.

$9G + 14B = 68$ (2)

I am trying to find a solution that gives me a value for B and G that satisfies both equations. What I am looking for is the **simultaneous** solution to these equations.

We have two equations involving two variables (G and B). This is enough to allow me to find the simultaneous solution.

There are a number of ways of finding the simultaneous solution and we will present only one of these in this module. The method we will use is called the **substitution method**.

The steps involved in finding a simultaneous solution using the substitution method are:

1. Using either of the equations, express one variable in terms of the other.
2. This expression is then **substituted** into the other equation to form an equation in one variable only.
3. Solve this equation to find the value of one of the variables.
4. Substitute the value of this variable into the equation formed in the first step to find the value of the other variable.
5. Finally, check your answer in **both the original equations**.

Let's follow these steps through for our Australian Rules dilemma.

$$17G + 1B = 103 \quad (1)$$

$$9G + 14B = 68 \quad (2)$$

Step 1. Using either of the equations, express one variable in terms of the other.

Since equation (1) has a variable on its own (with a coefficient of 1), that is the easiest equation to rearrange.

From (1)

$$\begin{aligned} 17G + 1B &= 103 \\ 1B &= 103 - 17G \\ B &= 103 - 17G \end{aligned}$$

Remember $1B$ is the same as B.

Step 2. Substitute this expression into the other equation to form an equation in one variable only.

The other equation is $9G + 14B = 68 \quad (2)$

So, everywhere we see a B we must substitute $103 - 17G$

That is, $9G + 14\,(103 - 17G) = 68$

Step 3. Solve this equation to find the value of one of the variables.

$$\begin{aligned} 9G + 14\,(103 - 17G) &= 68 \\ 9G + 14 \times 103 - 14 \times 17G &= 68 && \text{Using the distributive law.} \\ 9G + 1\,442 - 238G &= 68 \\ 9G - 238G + 1\,442 &= 68 && \text{Group like terms.} \\ (9 - 238)G + 1\,442 &= 68 \\ -229G + 1\,442 &= 68 \\ -229G + 1\,442 - 1\,442 &= 68 - 1\,442 \end{aligned}$$

$$-229G = -1\ 374$$

$$\frac{-229G}{-229} = \frac{-1\ 374}{-229}$$

$$G = 6$$

Step 4. Substitute the value of this variable into the equation formed in the first step to find the value of the other variable.

Substitute $G = 6$ into $B = 103 - 17G$

$$B = 103 - 17 \times 6$$

$$B = 103 - 102$$

$$B = 1$$

From our answers a goal scores 6 points and a behind scores 1 point.

Step 5. Check the answer in **both the original equations**.

Equation 1: LHS $= 17 \times 6 + 1 \times 1$

$= 102 + 1$

$= 103$

$=$ RHS

Equation 2: LHS $= 9 \times 6 + 14 \times 1$

$= 54 + 14$

$= 68$

$=$ RHS

The answer must be correct.

Example

At a local school production, adults were charged $5 entry and children $2. A total of 400 people attended for total receipts of $1 100. The school wishes to know how many of the people attending were adults, and how many children.

Before we are ready to solve the equations, following the steps set out before, we must firstly form the equations. As before, we must define the variables we are going to use.

Let A be the **number of adults** attending the production.

Let C be the **number of children** attending the production.

Note that we must be very specific about our variables. It is the **number** of adults and the **number** of children that we are interested in. We are not interested in their age or their sex.

We form the equations just as we have done in module 5 and earlier in this module.

If a total of 400 people attended the performance, then the number of adults plus the number of children equals 400.

$$A + C = 400 \qquad (1)$$

If each adult paid \$5 and each child \$2 then the total cost was \$1 100. The total cost paid by adults will be \$5 multiplied by the number of adults, and the total cost paid by children will be \$2 multiplied by the number of children.

$$5A + 2C = 1\ 100 \qquad (2)$$

Now we have the two equations we can go ahead and solve them as before.

From equation (1) $A + C = 400$ Subtract C from both sides.

$$A = 400 - C$$

Substitute into equation (2)

$$\begin{aligned}
5A + 2C &= 1\ 100 \\
5(400 - C) + 2C &= 1\ 100 \\
2\ 000 - 5C + 2C &= 1\ 100 \\
2\ 000 + -5C + 2C &= 1\ 100 \\
2\ 000 + (-5 + 2)C &= 1\ 100 \\
2\ 000 + -3C &= 1\ 100 \\
-3C &= 1\ 100 - 2\ 000 \\
-3C &= -900 \\
\frac{-3C}{-3} &= \frac{-900}{-3} \\
C &= 300
\end{aligned}$$

Substitute $C = 300$ into

$$\begin{aligned}
A &= 400 - C \\
A &= 400 - 300 \\
A &= 100
\end{aligned}$$

Therefore 300 children and 100 adults attended.

Checking these answers in the original equations.

Equation 1:

$$\begin{aligned}
\text{LHS} &= A + C \\
&= 100 + 300 \\
&= 400 \\
&= \text{RHS}
\end{aligned}$$

Equation 2:

$$\begin{aligned}
\text{LHS} &= 5A + 2C \\
&= 5 \times 100 + 2 \times 300 \\
&= 500 + 600 \\
&= 1\ 100 \\
&= \text{RHS}
\end{aligned}$$

The answer must be correct.

Activity

Activity 7.6

1. Solve the following sets of simultaneous equations.

   (a) $$\begin{aligned} y - x &= 0 \\ x + 2y &= 12 \end{aligned}$$

   (b) $$\begin{aligned} 3x + y &= 7 \\ 2x + y &= 5 \end{aligned}$$

   (c) $$\begin{aligned} 5x + y &= 15 \\ 3x - y &= 1 \end{aligned}$$

   (d) $$\begin{aligned} y + 4x &= 7 \\ x - 2y &= 4 \end{aligned}$$

   (e) $$\begin{aligned} 5x + 2y &= 3 \\ 2x + 3y &= -1 \end{aligned}$$

2. The second hand music shop, Loud and Lousy, sells all of its cassettes for one fixed price and all of its compact discs (CD's) for another fixed price. A cassette and compact disc together cost $17. However, two cassettes and three CD's cost $46. How much does each cassette and each CD cost?

3. Suzy Speedy finds that if she walks for three hours and cycles for two hours, she can cover 51 km. If on the other hand she cycles three hours and walks two hours, she can cover 69 km. What are Suzy Speedy's walking and cycling speeds?

4. Living on Anklebiter Avenue are 26 children. There are two more boys than there are girls. How many girls and how many boys live on Anklebiter Avenue?

5. Allan and Pei-shu have a combined income of $52 000 a year. If Allan earns $2 800 less than Pei-shu, how much does each person earn?

6. After a successful car wash the Black Stump Scout troupe had raised $115 all in one and two dollar coins. There are fourteen more $2 coins than $1 coins. How many of each did they have?

7. Solve the following sets of simultaneous equation in (a) and (b).

   (a) $$\begin{aligned} 4x - y &= 6 \\ 2x + 3y &= 17 \end{aligned}$$

   (b) $$\begin{aligned} 3x + 4y &= 6 \\ 5x + 3y &= 1 \end{aligned}$$

   (c) A shoe company sells all styles of handmade leather men's and ladies shoes for a set price. Last month they sold 50 pairs of men's shoes and 300 pairs of ladies shoes for a total of $47 500. This month sales have peaked and reached a high of $58 510 selling 61 pairs of men's and 370 pairs of ladies shoes. Determine the cost of a pair of ladies shoes and a pair of men's shoes.

7.6 A taste of things to come

1. Ask a person to write the number of the month of his or her birth and perform the following operations in order. Multiply by 5, add 6, multiply this answer by 4 and add 9 to the result. Now multiply the result by 5 and add the number of the day of birth. When 165 is subtracted from this number, the result is a number that represents the person's month and day of birth. Try it!

   (a) Now try to write out the above steps as an expression. You will need to use lots of brackets. Be sure to define your variables.

   (b) Simplify this expression and show that it equals $100m + d$

2. Equations are often used by many people to determine costs, revenue, profits and losses. Consider the following small company making silver bracelets. The daily costs, excluding labour, are the fixed costs (rent, electricity, etc.) of $100 and the cost of purchasing parts at $3 per bracelet.

   (a) What is the formula used to calculate the daily cost of producing bracelets? Define all variables.

   (b) Calculate the cost of producing the following bracelets per day.

      (i) 120

      (ii) 800

      (iii) 1 000

   (d) Rearrange the formula to derive a formula which gives the number of bracelets that can be produced in a day from a given expenditure. (That is, make the number of bracelets the subject of the formula.)

   (e) Use your formula from (c) to determine the number of bracelets that can be produced if the total daily costs are to be $1 600.

3. The amazing Emperor Penguins have the ability to dive to depths in excess of 450 metres to feed.

   Using the formula speed = $\frac{\text{distance}}{\text{time}}$, calculate the following:

   (a) If the rate of ascent and descent of a penguin's dive is 2.2 metres per second, express this speed in metres per minute.

   (b) The duration of the dive is approximately 10 minutes, with the penguin taking 3 minutes to feed in the ocean depths. How long does it take the penguin to reach the lowest point?

   (c) Find the depth of the dive to the nearest metre.

4. If you go on to study subjects in economics you will no doubt come across the concepts of supply and demand. Supply and demand for an item is based on price. The greater the price of an item the less it is in **demand** because people cannot afford to buy it. The greater the price though, the more of the item the manufacturer wants to **supply**, because they are getting a good price.

   At some point there has to be a compromise between the amount supplied and the amount demanded. This is called the **point of equilibrium.**

   Consider a small company making pottery bowls. They find that the equation for the quantity of bowls supplied is given by:

   $Q = 10P - 10$ where Q represents the quantity supplied.

   and P represents the price of each bowl in dollars.

   They also find that the equation for the quantity of bowls demanded is given by:

   $Q = -20P + 65$ where Q represents the quantity demanded.

   and P represents the price of each bowl in dollars.

   By solving these two equations simultaneously you will find the price of a bowl that will give the point of equilibrium between supply and demand.

   Go ahead now and solve these two equations simultaneously.

You should now be ready to attempt all questions of Assignment 4A (see your Introductory Book for details). If you have any questions, please refer them to your course tutor.

7.7 Post-test

1. Rearrange the following equations to make a the subject.

   (a) $T = \frac{ab}{2}$

   (b) $y = \sqrt{3.1a} + 2.4$

2. Solve the following equations, checking your solutions.

   (a) $4x + 3 = -2x - 9$

   (b) $x^2 - 9 = 0$

3. A rectangle has its width 3 cm shorter than its length. If the perimeter is 42 cm, find the width of the rectangle.

4. A ladder 2 metres long is placed with its top resting against a wall and its foot on the ground 80 centimetres from the wall. How high up the wall does the ladder reach? (Round your answer to two decimal places.)

5. Henry and Alice, trying to relive the 70's, went into a second hand record shop. Henry bought three records and two tapes for $25.50 and Alice bought two records and four tapes for $29.00. If each paid the same price for records and tapes, how much was one record and one tape.

7.8 Solutions

Solutions to activities

Activity 7.1

(a) Let the number be N.$N + 5 =$ 35

(b) Let the number be N $N - 9 = 2N$

(c) Let the number be N $6.5N = -3.4$

(d) Let the number be N $5N + 7 =$ 22

6.

(a) $B = 5J$ where B represents Ben's age in years,
and J represents John's age in years.

(b) $L = W + 7$ where L represents the length of the rectangle in metres,
and W represents the width of the rectangle in metres.

(c) $K = J - 5$ where K represents Katie's age in years,
and J represents Jack's age in years.

(d) $B = \frac{1}{3}H$ where B represents the amount of money that Barry earns,
and H represents the amount of money that Harry earns.

Activity 7.2

1. $3x$, $7x$, $17x$, $8x$, $-4x$ and $-2x$ are like terms

   $5a$, $9a$, $12a$ and $-3a$ are like terms

   $7a^2$, $6a^2$ and $2a^2$ are like terms

   $6x^2$, $5x^2$, $9x^2$ and $-11x^2$

2.

(a) $5x + 6x = (5 + 6)x = 11x$

(b) $6x^2 + 11x^2 = (6 + 11)x^2 = 17x^2$

(c) $x + 3x = (1 + 3)x = 4x$

(d) $7x - 3x = (7 - 3)x = 4x$

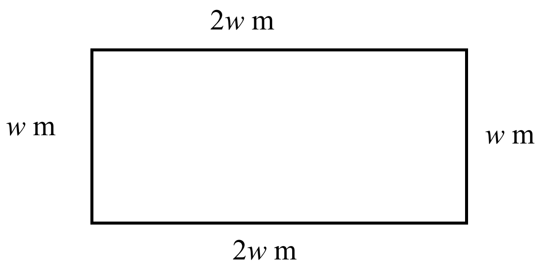
(e) $9x - 14x = (9 - 14)x = -5x$

(f) $-8x - 4x = -8x + -4x = (-8 + -4)x = -12x$

(g) $-3.1x + -2.4x = (-3.1 + -2.4)x = -5.5x$

(h) $4x + 32x - 7x = (4 + 32 - 7)x = 29x$

3. Firstly draw a diagram to represent the situation.



If the perimeter of the house is 72 metres, then the equation becomes:

$$w + 2w + w + 2w = 72$$

$$(1 + 2 + 1 + 2)w = 72$$

$$6w = 72$$

4. Again a diagram will help clarify the situation.

Let the length of the shorter piece of rope be L metres.

Together the pieces of rope must be 75 metres, so the equation becomes:

$$L + L + 15 = 75$$

$$(1 + 1)L + 15 = 75$$

$$2L + 15 = 75$$

Activity 7.3

1.

(a)

$$\begin{aligned} R &= 4a \\ \frac{R}{4} &= \frac{\cancel{4}a}{\cancel{4}} \\ \frac{R}{4} &= a \\ a &= \frac{R}{4} \end{aligned}$$

(b)

$$\begin{aligned} x + 2 &= y \\ x + 2 - 2 &= y - 2 \\ x &= y - 2 \end{aligned}$$

(c)

$$\begin{aligned} \frac{5}{b} &= k \\ \frac{5 \times \cancel{b}}{\cancel{b}} &= k \times b \\ 5 &= kb \\ \frac{5}{k} &= \frac{\cancel{k}b}{\cancel{k}} \\ \frac{5}{k} &= b \\ b &= \frac{5}{k} \end{aligned}$$

(d)

$$\begin{aligned} y &= 5x + 2 \\ y - 2 &= 5x + 2 - 2 \\ y - 2 &= 5x \\ \frac{y-2}{5} &= \frac{5x}{5} \\ \frac{y-2}{5} &= x \\ x &= \frac{y-2}{5} \end{aligned}$$

(e)

$$\begin{aligned} 7x - 8 &= y \\ 7x - 8 + 8 &= y + 8 \\ 7x &= y + 8 \\ \frac{\cancel{7}x}{\cancel{7}} &= \frac{y+8}{7} \\ x &= \frac{y+8}{7} \end{aligned}$$

(f)

$$\begin{aligned} \frac{6}{t} + 7 &= s \\ \frac{6}{t} + 7 - 7 &= s - 7 \\ \frac{6}{t} &= s - 7 \\ \frac{6}{\cancel{t}} \times \cancel{t} &= (s-7)t \\ 6 &= (s-7)t \\ \frac{6}{(s-7)} &= \frac{\cancel{(s-7)}t}{\cancel{(s-7)}} \\ \frac{6}{(s-7)} &= t \\ t &= \frac{6}{(s-7)} \end{aligned}$$

(g)

$$\begin{aligned}
\frac{x+8}{3} &= y \\
\frac{x+8}{\cancel{3}} \times \cancel{3} &= y \times 3 \\
x+8 &= y \times 3 \\
x+8 &= 3y \\
x+8-8 &= 3y-8 \\
x &= 3y-8
\end{aligned}$$

2. where P represents the perimeter, and s represents the **length** of one side.

$$\begin{aligned}
P &= 4s \\
\frac{P}{4} &= \frac{\cancel{4}s}{\cancel{4}} \\
\frac{P}{4} &= s \\
s &= \frac{P}{4}
\end{aligned}$$

3.

$$\begin{aligned}
h &= 10-2t \\
h-10 &= 10-2t-10 \\
h-10 &= -2t \\
\frac{h-10}{-2} &= \frac{\cancel{-2}t}{\cancel{-2}} \\
\frac{h-10}{-2} &= t \\
t &= \frac{h-10}{-2}
\end{aligned}$$

4.

$$\begin{aligned}
m &= 3.5 + 2.5x \\
m-3.5 &= 3.5+2.5x-3.5 \\
m-3.5 &= 2.5x \\
\frac{m-3.5}{2.5} &= \frac{\cancel{2.5}x}{\cancel{2.5}} \\
\frac{m-3.5}{2.5} &= x \\
x &= \frac{m-3.5}{2.5}
\end{aligned}$$

5.

$$\begin{aligned}
C &= \pi d \\
\frac{C}{\pi} &= \frac{\cancel{\pi}d}{\cancel{\pi}} \\
\frac{C}{\pi} &= d \\
d &= \frac{C}{\pi}
\end{aligned}$$

Activity 7.4

1.

(a)

$$
\begin{aligned}
x + 3 &= 9 \\
x + 3 - 3 &= 9 - 3 \qquad \text{Isolate } x \text{ by taking 3 from both sides.} \\
x &= 6
\end{aligned}
$$

$$
\begin{aligned}
\text{Check:} \quad \text{LHS} &= 6 + 3 \\
&= 9 \\
&= \text{RHS}
\end{aligned}
$$

(b)

$$
\begin{aligned}
x - 2 &= 5 \\
x - 2 + 2 &= 5 + 2 \qquad \text{Isolate } x \text{ by adding 2 to both sides.} \\
x &= 7
\end{aligned}
$$

$$
\begin{aligned}
\text{Check:} \quad \text{LHS} &= 7 - 2 \\
&= 5 \\
&= \text{RHS}
\end{aligned}
$$

(c)

$$
\begin{aligned}
5x &= 15 \qquad \text{Isolate } x \text{ by dividing both sides by 5.} \\
\frac{5x}{\not 5} &= \frac{15}{5} \\
x &= 3
\end{aligned}
$$

$$
\begin{aligned}
\text{Check:} \quad \text{LHS} &= 5 \times 3 \\
&= 15 \\
&= \text{RHS}
\end{aligned}
$$

(d)

$$
\begin{aligned}
\frac{x}{2} &= 6 \qquad \text{Isolate } x \text{ by multiplying both sides by 2.} \\
\frac{x}{\not 2} \times \not 2 &= 6 \times 2 \\
x &= 12
\end{aligned}
$$

$$
\begin{aligned}
\text{Check:} \quad \text{LHS} &= \frac{12}{2} \\
&= 6 \\
&= \text{RHS}
\end{aligned}
$$

(e) $-3 + x = 7$ Isolate x by adding 3 to both sides.

$$\begin{aligned} -3 + x + 3 &= 7 + 3 \\ x &= 10 \end{aligned}$$

Check: $$\begin{aligned} \text{LHS} &= -3 + 10 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

(f) $2.4 - x = 3.6$ Isolate x by taking 2.4 from both sides.

$$2.4 - x - 2.4 = 3.6 - 2.4$$

$$-x = 1.2$$ Multiply both sides by –1 to make coefficient of the x a 1.

$$x = -1.2$$

Check: $$\begin{aligned} \text{LHS} &= 2.4 - (-1.2) \\ &= 2.4 + 1.2 \\ &= 3.6 \\ &= \text{RHS} \end{aligned}$$

(g) $2x + 1 = -8$ Isolate x by taking 1 from both sides.

$$2x + 1 - 1 = -8 - 1$$

$$2x = -9$$ Divide both sides by 2 to make coefficient of x a one.

$$x = \frac{-9}{2}$$

Check: $$\begin{aligned} \text{LHS} &= 2x + 1 \\ &= \cancel{2} \times \frac{-9}{\cancel{2}} + 1 \\ &= -9 + 1 \\ &= -8 \\ &= \text{RHS} \end{aligned}$$

(h) $\frac{x}{5} - 2 = 7$ Isolate x by adding 2 to both sides.

$\frac{x}{5} - 2 + 2 = 7 + 2$

$\frac{x}{5} = 9$ Multiply both sides by 5.

$x = 45$

Check: LHS $= \frac{45}{5} - 2$

$= 9 - 2$

$= 7$

$=$ RHS

(i) $5 + 2x = 1$ Take 5 from both sides to isolate the x.

$5 + 2x - 5 = 1 - 5$

$2x = -4$ Divide both sides by 2 to make coefficient of x a one.

$x = -2$

Check: LHS $= 5 + 2 \times -2$

$= 5 - 4$

$= 1$

$=$ RHS

(j) $10 - 3x = 1$ Take 10 from both sides to isolate the x.

$10 - 3x - 10 = 1 - 10$

$-3x = -9$ Divide both sides by -3 to make coefficient of x a one.

$x = 3$

Check: LHS $= 10 - 3 \times 3$

$= 10 - 9$

$= 1$

$=$ RHS

(k)
$$\frac{6}{t} + 2 = 12$$
$$\frac{6}{t} + 2 - 2 = 12 - 2$$
$$\frac{6}{t} = 10$$
$$\frac{6 \times \not{t}}{\not{t}} = 10 \times t$$
$$6 = 10t$$
$$\frac{6}{10} = \frac{10t}{10}$$
$$\frac{6}{10} = t$$
$$t = \frac{6}{10}$$
$$t = 0.6$$

Check: $\text{LHS} = \frac{6}{t} + 2$

$= \frac{6}{0.6} + 2$

$= 10 + 2$

$= 12$

$= \text{RHS}$

(l) $2.5x + 3.5 = 7$ Subtract 3.5 from both sides.

$2.5x + 3.5 - 3.5 = 7 - 3.5$

$2.5x = 3.5$ Divide both sides by 2.5.

$x = 1.4$

Check: $\text{LHS} = 2.5 \times 1.4 + 3.5$

$= 3.5 + 3.5$

$= 7$

$= \text{RHS}$

(m) $4(3x - \frac{4}{9}) = 3$

$$\frac{\not{4}\left(3x - \frac{4}{9}\right)}{\not{4}} = \frac{3}{4}$$

$$3x - \frac{4}{9} = \frac{3}{4}$$

$$3x - \frac{4}{9} + \frac{4}{9} = \frac{3}{4} + \frac{4}{9}$$

$$3x = \frac{3}{4} + \frac{4}{9}$$

$$3x = \frac{27}{36} + \frac{16}{36}$$

$$3x = \frac{43}{36}$$

$$3x \div 3 = \frac{43}{36} \div 3$$

$$x = \frac{43}{36} \times \frac{1}{3}$$

$$x = \frac{43}{108}$$

Check: LHS $= 4(3x - \frac{4}{9})$

$$= 4\left(3 \times \frac{43}{108} - \frac{4}{9}\right)$$

$$= 4\left(\frac{43}{36} - \frac{4}{9}\right)$$

$$= 4\left(\frac{43}{36} - \frac{16}{36}\right)$$

$$= 4 \times \frac{27}{36}$$

$$= 3$$

= RHS

(n)
$$\begin{aligned}
2x + 3 + 5x &= 17 \quad \text{Group like terms}\\
2x + 5x + 3 &= 17\\
(2 + 5)x + 3 &= 17\\
7x + 3 &= 17\\
7x + 3 - 3 &= 17 - 3\\
7x &= 14\\
x &= 2
\end{aligned}$$

Check:
$$\begin{aligned}
\text{LHS} &= 2 \times 2 + 3 + 5 \times 2\\
&= 4 + 3 + 10\\
&= 17\\
&= \text{RHS}
\end{aligned}$$

(o)
$$\begin{aligned}
5x + 3 - 2x &= 21\\
5x - 2x + 3 &= 21\\
3x + 3 &= 21\\
3x + 3 - 3 &= 21 - 3\\
3x &= 18\\
x &= 6
\end{aligned}$$

Check:
$$\begin{aligned}
\text{LHS} &= 5 \times 6 + 3 - 2 \times 6\\
&= 30 + 3 - 12\\
&= 21\\
&= \text{RHS}
\end{aligned}$$

(p)
$$\begin{aligned}
2x + 1 &= 3x - 5\\
2x + 1 - 3x &= 3x - 5 - 3x\\
2x - 3x + 1 &= -5\\
-x + 1 &= -5\\
-x + 1 - 1 &= -5 - 1\\
-x &= -6\\
x &= 6
\end{aligned}$$

Check:
$$\begin{aligned}
\text{LHS} &= 2 \times 6 + 1 & \text{RHS} &= 3 \times 6 - 5\\
&= 12 + 1 & &= 18 - 5\\
&= 13 & &= 13\\
& & &= \text{LHS}
\end{aligned}$$

(q)

$$
\begin{aligned}
0.5x - 4.1 &= 3.9 - 0.7x \\
0.5x - 4.1 + 0.7x &= 3.9 - 0.7x + 0.7x \\
0.5x + 0.7x - 4.1 &= 3.9 \\
1.2x - 4.1 &= 3.9 \\
1.2x - 4.1 + 4.1 &= 3.9 + 4.1 \\
1.2x &= 8 \\
\frac{\not{1.2}x}{\not{1.2}} &= \frac{8}{1.2} \\
x &= \frac{80}{12} = \frac{20}{3} = 6\ \frac{2}{3}
\end{aligned}
$$

Check:

$$
\begin{aligned}
\text{LHS} &= 0.5 \times \frac{20}{3} - 4.1 & \text{RHS} &= 3.9 - 0.7 \times \frac{20}{3} \\
&= 3.333... - 4.1 & &= 3.9 - 4.666... \\
&\approx -0.7667 & &\approx -0.7667 \\
& & &= \text{LHS}
\end{aligned}
$$

2. If the perimeter of the house is 72 metres, then the equation becomes:

$$
\begin{aligned}
w + 2w + w + 2w &= 72 \\
(1 + 2 + 1 + 2)w &= 72 \\
6w &= 72 \\
\frac{6w}{6} &= \frac{72}{6} \\
w &= 12
\end{aligned}
$$

The width of the house is 12 metres so the length of the house must be 24 metres.

Check: the perimeter of the house 12 m by 24 m is 12 + 24 + 12 + 24 = 72 which is what we started with.

3. Let the length of the shorter piece of rope be L metres.

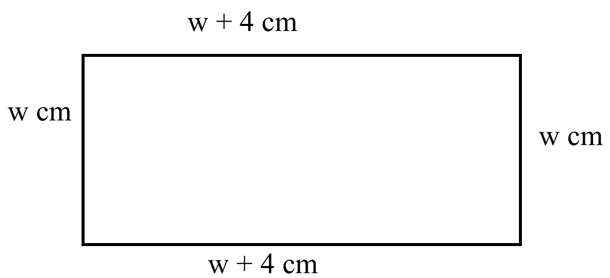
Together the pieces of rope must be 63 metres, so the equation becomes:

$$
\begin{aligned}
L + L + 17 &= 63 \\
(1 + 1)L + 17 &= 63 \\
2L + 17 &= 63 \\
2L + 17 - 17 &= 63 - 17 \\
2L &= 46 \\
\frac{2L}{2} &= \frac{46}{2} \\
L &= 23
\end{aligned}
$$

The two pieces of rope are 23 metres and 40 metres.

Check: 23 m + 40 m = 63 m which is the length of rope we started with.

4. Let the width of the rectangle be w cm. The length will be $w + 4$ cm.



The perimeter is given by the equation:

$$
\begin{aligned}
w + 4 \ + \ w \ + w + 4 \ + \ w &= 52 \\
1w + 1w + 1w + 1w + 4 + 4 &= 52 \\
(1 + 1 + 1 + 1)w + (4 + 4) &= 52 \quad \text{Using the distributive law} \\
4w + 8 &= 52 \quad \text{Simplifying.} \\
4w + 8 - 8 &= 52 - 8 \\
4w &= 44
\end{aligned}
$$

$$\frac{4w}{4} = \frac{44}{4}$$

$$w = 11$$

The width of the rectangle is 11 cm and the length is 15 cm.

Check: The perimeter of this rectangle is 11 + 15 + 11 + 15 = 52 cm which is the perimeter we started with.

5. Let the number thought of be *N*. The equation becomes:

$$2N + 7 = 59$$

$$2N + 7 - 7 = 59 - 7$$

$$2N = 52$$

$$\frac{2N}{2} = \frac{52}{2}$$

$$N = 26$$

The number thought of was 26.

Check: think of a number, 26, double it, 52, add 7, 59. The result is 59 as it should have been.

6. Let the number be *N*. The equation becomes:

$$\frac{1}{3} \times N + 23 = 34$$

$$\frac{N}{3} + 23 = 34$$

$$\frac{N}{3} + 23 - 23 = 34 - 23$$

$$\frac{N}{3} = 11$$

$$\frac{N}{3} \times 3 = 11 \times 3$$

$$N = 33$$

The number was 33.

Check: One third of a number, 11, added to 23, gives 34, which is the required answer.

7. Let Rebecca's age be R years. The equation becomes:

$$
\begin{aligned}
\frac{2R-3}{5} &= 5 \\
\frac{2R-3}{\not 5} \times \not 5 &= 5 \times 5 \\
2R-3 &= 25 \\
2R-3+3 &= 25+3 \\
2R &= 28 \\
\frac{\not 2 R}{\not 2} &= \frac{28}{2} \\
R &= 14
\end{aligned}
$$

Therefore, Rebecca is 14 years old.

Check: Double Rebecca's age, 28, subtract three, 25, and divide the result by 5, giving 5 as expected.

8. (a) $P = 50 - 7.75x$ where P represents the amount of money (in dollars) left, and x represents the number of sample bags bought.

(b) If $P = 19$, solve the equation for x.

$$
\begin{aligned}
19 &= 50 - 7.75x \\
19 - 50 &= 50 - 7.75x - 50 \\
-31 &= -7.75x \\
\frac{-31}{-7.75} &= \frac{-7.75x}{-7.75} \\
4 &= x
\end{aligned}
$$

Samantha bought 4 sample bags with the money.

Check:

$$
\begin{aligned}
\text{RHS} &= 50 - 7.75x \\
&= 50 - 7.75 \times 4 \\
&= 50 - 31 \\
&= 19 \\
&= \text{LHS}
\end{aligned}
$$

9. Let the number of children bought for be x.

The cost of the chips is $1.2x$ and the sweets 2.5 times this at $2.5 \times 1.2x = 3x$

The equation to represent Mrs Manykids' purchases is:

$$1.2x + 3x = 42$$

$$(1.2 + 3)x = 42$$

$$4.2x = 42$$

$$\frac{4.2x}{4.2} = \frac{42}{4,2}$$

$$x = 10$$

Mrs Manykids bought for 10 children.

Check:

$$\begin{aligned} \text{LHS} &= 1.2x + 3x \\ &= 1.2 \times 10 + 3 \times 10 \\ &= 12 + 30 \\ &= 42 \\ &= \text{RHS} \end{aligned}$$

10. (a)

$$\frac{2}{5} + \frac{7}{3}t = 5$$

$$\frac{7}{3}t = 5 - \frac{2}{5} = \frac{25}{5} - \frac{2}{5}$$

$$\frac{7}{3}t = \frac{23}{5}$$

$$7t = \frac{23}{5} \times 3$$

$$7t = \frac{69}{5}$$

$$t = \frac{69}{5 \times 7}$$

$$t = \frac{69}{35}$$

(b)

$$\begin{aligned}
\frac{2}{3}x + 7 &= \frac{1}{2}x - 3 \\
\frac{2}{3}x - \frac{1}{2}x &= -3 - 7 \\
\frac{4}{6}x - \frac{3}{6}x &= -10 \\
\frac{1}{6}x &= -10 \\
x &= -10 \times 6 \\
x &= -60
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{7}{8} - \frac{3}{8}x + 4 &= \frac{3}{4} + 2x \\
\frac{7}{8} - \frac{3}{4} + 4 &= 2x + \frac{3}{8}x \\
\frac{7}{8} - \frac{6}{8} + \frac{32}{8} &= \frac{16}{8}x + \frac{3}{8}x \\
\frac{33}{8} &= \frac{19}{8}x \\
33 &= 19x \\
x &= \frac{33}{19}
\end{aligned}$$

Activity 7.5

1.

(a)

$$\begin{aligned}
b &= 2a^2 + 1 \\
b - 1 &= 2a^2 \\
\frac{b-1}{2} &= \frac{2a^2}{2} \\
\frac{b-1}{2} &= a^2 \\
a^2 &= \frac{b-1}{2} \\
a &= \pm\sqrt{\frac{b-1}{2}}
\end{aligned}$$

(b)

$$c = 2(3a-7)^2$$

$$\frac{c}{2} = \frac{2(3a-7)^2}{2}$$

$$\frac{c}{2} = (3a-7)^2$$

$$\pm\sqrt{\frac{c}{2}} = \sqrt{(3a-7)^2}$$

$$\pm\sqrt{\frac{c}{2}} = 3a-7$$

$$\pm\sqrt{\frac{c}{2}} + 7 = 3a$$

$$\frac{\pm\sqrt{\frac{c}{2}} + 7}{3} = a$$

(c)

$$T = \sqrt{1.5-2.6a}$$

$$T^2 = (\sqrt{1.5-2.6a})^2$$

$$T^2 = 1.5-2.6a$$

$$T^2 - 1.5 = -2.6a$$

$$\frac{T^2-1.5}{-2.6} = \frac{\cancel{-2.6}a}{\cancel{-2.6}}$$

$$\frac{T^2-1.5}{-2.6} = a$$

$$a = \frac{T^2-1.5}{-2.6}$$

(d)

$$m = \frac{\sqrt{a^2+2}}{5}$$

$$5m = \sqrt{a^2+2}$$

$$(5m)^2 = \left(\sqrt{a^2+2}\right)^2$$

$$(5m)^2 = a^2+2$$

$$(5m)^2 - 2 = a^2$$

$$a^2 = (5m)^2 - 2$$

$$a = \pm\sqrt{(5m)^2 - 2}$$

(e)

$$\text{B} = 6\sqrt{3a+7}$$

$$\frac{\text{B}}{6} = \sqrt{3a+7}$$

$$\left(\frac{\text{B}}{6}\right)^2 = 3a+7$$

$$\left(\frac{\text{B}}{6}\right)^2 - 7 = 3a$$

$$\frac{\left(\frac{\text{B}}{6}\right)^2 - 7}{3} = a$$

2.

(a)

$$S = 4.9\,t^2$$

$$\frac{S}{4.9} = t^2$$

$$t^2 = \frac{\text{S}}{4.9}$$

$$\text{t} = \sqrt{\frac{\text{S}}{4.9}}$$

We can ignore the negative sign this time as a negative time has no meaning.

(b) When $S = 650$ m then:

$$t = \sqrt{\frac{S}{4.9}}$$

$$t = \sqrt{\frac{650}{4.9}}$$

$$t = \sqrt{132.65306....}$$

$$t \approx 11.5$$

When the cliff is 650 metres high, it would take about 11.5 seconds for a stone to reach the ground.

3.

(a)

$$D = 8\sqrt{\left(\frac{h}{5}\right)}$$

$$\frac{D}{8} = \sqrt{\left(\frac{h}{5}\right)}$$

$$\left(\frac{D}{8}\right)^2 = \left(\frac{h}{5}\right)$$

$$5\left(\frac{D}{8}\right)^2 = h$$

$$h = 5\left(\frac{D}{8}\right)^2$$

(b)

$$h = 5\left(\frac{D}{8}\right)^2$$

$$h = 5\left(\frac{15}{8}\right)^2$$

$$h = 5 \times 3.515625$$

$$h = 17.578125$$

$$h \approx 18$$

If a distance of fifteen kilometres can be seen from the tower, it must be about 18 metres tall.

4.

Let the number I thought of be N.

Then, $4N^2 = 36$

$$\frac{4N^2}{4} = \frac{36}{4}$$

$$N^2 = 9$$

$$N = \pm 3$$

In fact there are two numbers that satisfy this question. The number could have been 3 or –3.

For all of the following questions the negative square root is ignored as it has no meaning in the context of these questions.

5.

(a)

$x^2 = 4^2 + 3^2$

$x^2 = 16 + 9$

$x^2 = 25$

$x = \sqrt{25}$

$x = 5$

Unknown length = 5 cm

(b)

$(10.6)^2 = (3.7)^2 + x^2$

$112.36 = 13.69 + x^2$

$x^2 = 112.36 - 13.69$

$x^2 = 98.67$

$x = \sqrt{98.67}$

$x \approx 9.93$

Unknown length $\approx$ 9.93 m

(c)

$96^2 = 57^2 + x^2$

$9\,216 = 3\,249 + x^2$

$x^2 = 9\,216 - 3\,249$

$x^2 = 5\,967$

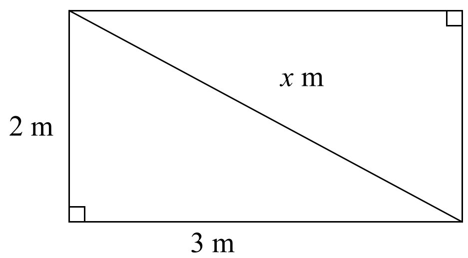
$x = \sqrt{5\,967}$

$x \approx 77.25$

Unknown length $\approx$ 77.25 mm

6.

Using Pythagoras' theorem



$x^2 = 2^2 + 3^2$

$x^2 = 4 + 9$

$x^2 = 13$

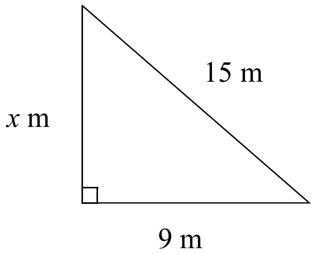
$x = \sqrt{13}$

$x \approx 3.606$

The brace should be 3.606 m to the nearest millimetre (3 606 mm).

7. Using Pythagoras' Theorem

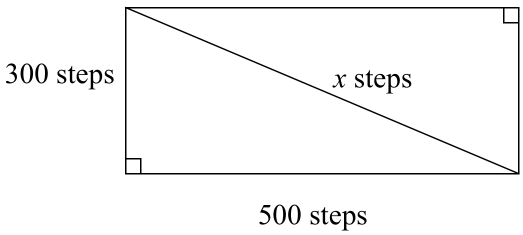
$$15^2 = 9^2 + x^2$$
$$225 = 81 + x^2$$
$$x^2 = 225 - 81$$
$$x^2 = 144$$
$$x = \sqrt{144}$$
$$x = 12$$



The wire reaches 12 metres up the pole.

8. Firstly calculate the number of steps across the field from corner to corner.

$$x^2 = 300^2 + 500^2$$
$$x^2 = 90\,000 + 250\,000$$
$$x^2 = 340\,000$$
$$x = \sqrt{340\,000}$$
$$x \approx 583$$



The distance across the field from corner to corner is approximately 583 steps.

To walk all the way round the field Joe takes 500 + 300 = 800 steps.

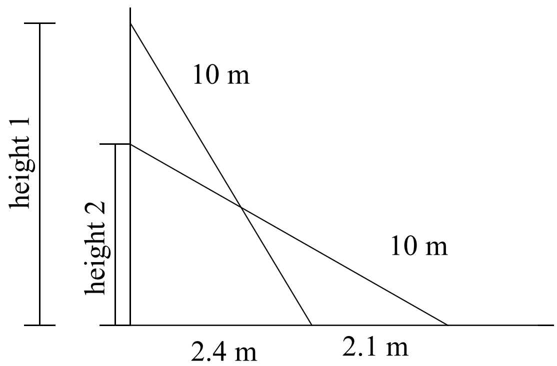
By cutting across the field he saves 800 – 583 = 217 steps.

9. A diagram will help clarify the question.

We can find the height up the wall of the ladder with the foot at 2.4 metres from the wall, and then find the height of the ladder after moving the foot out further from the wall. If we subtract these heights we will find the distance the ladder has moved down the wall. That is, height 1 – height 2 will give us the change in height of the ladder.

To find height 1:

$$10^2 = (2.4)^2 + (\text{height } 1)^2$$
$$100 = 5.76 + (\text{height } 1)^2$$
$$(\text{height } 1)^2 = 100 - 5.76$$
$$(\text{height } 1)^2 = 94.24$$
$$\text{height } 1 = \sqrt{94.24}$$
$$\text{height } 1 = 9.70772888\ldots$$



To find height 2:

$$
\begin{aligned}
10^2 &= (4.5)^2 + (\text{height } 2)^2 \\
100 &= 20.25 + (\text{height } 2)^2 \\
(\text{height } 2)^2 &= 100 - 20.25 \\
(\text{height } 2)^2 &= 79.75 \\
\text{height } 2 &= \sqrt{79.75} \\
\text{height } 2 &= 8.93028555\ldots
\end{aligned}
$$

$$
\begin{aligned}
\text{The change in distance up the wall} &= \text{height } 1 - \text{height } 2 \\
&= 9.70772888 - 8.93028555 \\
&\approx 0.8
\end{aligned}
$$

Therefore the ladder moves 0.8 m down the side of the house.

10. (a)

$$
\begin{aligned}
l &= \frac{2}{3}p^2 + 4 \\
l - 4 &= \frac{2}{3}p^2 \\
3(l-4) &= 2p^2 \\
\frac{3}{2}(l-4) &= p^{2}\text{'}me \\
\therefore p &= \pm\sqrt{\frac{3}{2}(l-4)}
\end{aligned}
$$

(b)

$$l = \frac{6\sqrt{3p - \frac{3}{4}}}{4}$$

$$4l = 6\sqrt{3p - \frac{3}{4}}$$

$$\frac{4l}{6} = \sqrt{3p - \frac{3}{4}}$$

$$\left(\frac{2l}{3}\right)^2 = 3p - \frac{3}{4}$$

$$\left(\frac{2l}{3}\right)^2 + \frac{3}{4} = 3p$$

$$\therefore p = \left(\frac{2l}{3}\right)^2 + \frac{3}{4}$$

$$p = \frac{4l^2}{27} + \frac{3}{12}$$

$$p = \frac{4l^2}{27} + \frac{1}{4}$$

(c)

$$l = -6\left(\frac{5}{3}p^2 + 8\right)$$

$$\frac{l}{-6} = \frac{5}{3}p^2 + 8$$

$$\frac{l}{-6} - 8 = \frac{5}{3}p^2$$

$$\frac{3}{5}\left(\frac{l}{-6} - 8\right) = p^2$$

$$p = \pm\sqrt{\frac{3}{5}\left(\frac{l}{-6} - 8\right)}$$

Activity 7.6

1. There are many variations on these answers. As long as your answer is correct and you have shown all the working you are probably correct.

(a)

$$y - x = 0 \qquad (1)$$

$$x + 2y = 12 \qquad (2)$$

From equation (1) $y = x$ (3)

Substitute this value for y into equation (2)

$$x + 2x = 12$$

$$3x = 12$$

$$x = 4$$

Substituting into equation (3)

$$y = x$$

$$y = 4$$

Therefore the solution is $x = 4$ and $y = 4$

Check: Equation (1)

$$\begin{aligned} \text{LHS} &= y - x \\ &= 4 - 4 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

Equation (2)

$$\begin{aligned} \text{LHS} &= x + 2y \\ &= 4 + 2 \times 4 \\ &= 4 + 8 \\ &= 12 \\ &= \text{RHS} \end{aligned}$$

(b)

$$3x + y = 7 \qquad (1)$$

$$2x + y = 5 \qquad (2)$$

From equation (1) $y = 7 - 3x \qquad (3)$

Substitute this value for y into equation (2)

$$\begin{aligned} 2x + y &= 5 \\ 2x + 7 - 3x &= 5 \\ 2x - 3x + 7 &= 5 \\ -x &= -2 \qquad \text{Multiply both sides by } -1 \\ x &= 2 \end{aligned}$$

Substituting into equation (3)

$$\begin{aligned} y &= 7 - 3x \\ y &= 7 - 3 \times 2 \\ y &= 7 - 6 \\ y &= 1 \end{aligned}$$

Therefore the solution is $x = 2$ and $y = 1$

Check: Equation (1)

$$\begin{aligned} \text{LHS} &= 3x + y \\ &= 3 \times 2 + 1 \\ &= 6 + 1 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

Equation (2)

$$\begin{aligned} \text{LHS} &= 2x + y \\ &= 2 \times 2 + 1 \\ &= 4 + 1 \\ &= 5 \\ &= \text{RHS} \end{aligned}$$

(c)

$$5x + y = 15 \qquad (1)$$

$$3x - y = 1 \qquad (2)$$

From equation (1) $y = 15 - 5x \qquad (3)$

Substitute this value for y into equation (2)

$$\begin{aligned} 3x - y &= 1 \\ 3x - (15 - 5x) &= 1 \end{aligned}$$

Take great care with negatives and the distributive law.

$$\begin{aligned} 3x - 1\,(15 - 5x) &= 1 \\ 3x - 1 \times 15 - {}^{-}1 \times 5x &= 1 \\ 3x - 15 + 5x &= 1 \\ 8x - 15 &= 1 \\ 8x &= 16 \\ x &= 2 \end{aligned}$$

Substituting into equation (3)

$$\begin{aligned} y &= 15 - 5x \\ y &= 15 - 5 \times 2 \\ y &= 15 - 10 \\ y &= 5 \end{aligned}$$

Therefore the solution is $x = 2$ and $y = 5$

Check: Equation (1) LHS

$$\begin{aligned} &= 5x + y \\ &= 5 \times 2 + 5 \\ &= 10 + 5 \\ &= 15 \\ &= \text{RHS} \end{aligned}$$

Equation (2) LHS

$$\begin{aligned} &= 3x - y \\ &= 3 \times 2 - 5 \\ &= 6 - 5 \\ &= 1 \end{aligned}$$

(d)

$$\begin{aligned} y + 4x &= 7 \qquad (1) \\ x - 2y &= 4 \qquad (2) \end{aligned}$$

From equation (2) $x = 4 + 2y$ (3)

Substitute this value for y into equation (1)

$$\begin{aligned} y + 4x &= 7 \\ y + 4(4 + 2y) &= 7 \\ y + 16 + 8y &= 7 \\ 9y + 16 &= 7 \\ 9y &= -9 \\ y &= -1 \end{aligned}$$

Substituting into equation (3)

$$\begin{aligned} x &= 4 + 2y \\ x &= 4 + 2 \times -1 \\ x &= 4 - 2 \\ x &= 2 \end{aligned}$$

Therefore the solution is $x = 2$ and $y = -1$

Check: Equation (1) LHS

$$\begin{aligned} &= y + 4x \\ &= -1 + 4 \times 2 \\ &= -1 + 8 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

Equation (2) LHS

$$\begin{aligned} &= x - 2y \\ &= 2 - 2 \times -1 \\ &= 2 + 2 \\ &= 4 \\ &= \text{RHS} \end{aligned}$$

(e)

$$5x + 2y = 3 \qquad (1)$$

$$2x + 3y = -1 \qquad (2)$$

From equation (1) $2y = 3 - 5x$

$$y = \frac{3-5x}{2} \qquad (3)$$

Substitute this value for y into equation (2)

$$2x + 3y = -1$$

$$2x + 3\,\frac{(3-5x)}{2} = -1$$

$$2x + \frac{9-15x}{2} = -1$$ Removing the bracket using the distributive law.

$$\frac{9-15x}{2} = -1 - 2x$$

$$\frac{(9-15x)}{\not{2}} \times \not{2} = (-1-2x) \times 2$$ Multiply both sides by 2.

$$9 - 15x = -2 - 4x$$

$$9 = -2 - 4x + 15x$$

$$9 = -2 + 11x$$

$$9 + 2 = 11x$$

$$11 = 11x$$

$$1 = x$$

Substituting into equation (3)

$$y = \frac{3-5x}{2}$$

$$y = \frac{3 - 5 \times 1}{2}$$

$$y = \frac{3-5}{2}$$

$$y = \frac{-2}{2}$$

$$y = -1$$

Therefore the solution is $x = 1$ and $y = -1$

Equation (1)

$$\begin{aligned} \text{LHS} &= 5x + 2y \\ &= 5 \times 1 + 2 \times {}^{-}1 \\ &= 5 - 2 \\ &= 3 \\ &= \text{RHS} \end{aligned}$$

Equation (2)

$$\begin{aligned} \text{LHS} &= 2x + 3y \\ &= 2 \times 1 + 3 \times -1 \\ &= 2 - 3 \\ &= -1 \\ &= \text{RHS} \end{aligned}$$

2. Let cassettes cost C dollars and let CD's cost D dollars.

$$\begin{aligned} C + D &= 17 && (1) \\ 2C + 3D &= 46 && (2) \\ \text{From (1)} \quad C &= 17 - D && (3) \end{aligned}$$

Substitute this value for C into equation (2)

$$\begin{aligned} 2C + 3D &= 46 \\ 2 \times (17 - D) + 3D &= 46 \\ 34 - 2D + 3D &= 46 \\ 34 + 1D &= 46 \\ D &= 46 - 34 \\ D &= 12 \end{aligned}$$

Substituting this value for D into equation (3)

$$\begin{aligned} C &= 17 - D \\ C &= 17 - 12 \\ C &= 5 \end{aligned}$$

The cost of a cassette is \$5 and the cost of a CD is \$12

Check:

Equation (1)

$$\begin{aligned} \text{LHS} &= C + D \\ &= 5 + 12 \\ &= 17 \\ &= \text{RHS} \end{aligned}$$

Equation (2)

$$\begin{aligned} \text{LHS} &= 2C + 3D \\ &= 2 \times 5 + 3 \times 12 \\ &= 10 + 36 \\ &= 46 \\ &= \text{RHS} \end{aligned}$$

3. The question is what gives you the clue about what to let the variables be. The question in this case is 'What are Suzy Speedy's walking and cycling speeds?'

   We will let Suzy's walking speed be W km/h

   and Suzy's cycling speed be C km/h.

$$3W + 2C = 51 \qquad (1)$$

$$3C + 2W = 69 \qquad (2)$$

From (1)

$$2C = 51 - 3W$$

$$C = \frac{51 - 3W}{2} \qquad (3)$$

Substitute this value for C into equation (2).

$$3C + 2W = 69$$

$$\frac{3(51 - 3W)}{2} + 2W = 69$$

$$\frac{153 - 9W}{2} + 2W = 69$$

$$\frac{153 - 9W}{2} = 69 - 2W$$

$$\frac{(153 - 9W)}{\not{2}} \times \not{2} = (69 - 2W) \times 2$$

$$153 - 9W = 138 - 4W$$

$$153 - 9W + 4W = 138 - 4W + 4W$$

$$153 - 5W = 138$$

$$153 - 5W - 153 = 138 - 153$$

$$-5W = -15$$

$$W = 3$$

Substitute this value for W into equation (3)

$$C = \frac{51 - 3W}{2}$$

$$C = \frac{51 - 3 \times 3}{2}$$

$$C = \frac{51 - 9}{2}$$

$$C = \frac{42}{2}$$

$$C = 21$$

Suzy's cycling speed is 21 km/hour and Suzy's walking speed is 3 km/hour.

Check:

Equation (1)

$$\begin{aligned} \text{LHS} &= 3W + 2C \\ &= 3 \times 3 + 2 \times 21 \\ &= 9 + 42 \\ &= 51 \\ &= \text{RHS} \end{aligned}$$

Equation (2)

$$\begin{aligned} \text{LHS} &= 3C + 2W \\ &= 3 \times 21 + 2 \times 3 \\ &= 63 + 6 \\ &= 69 \\ &= \text{RHS} \end{aligned}$$

4. Let the number of boys be *B* and the number of girls be *G*.

$$B + G = 26 \qquad (1)$$

$$B = G + 2 \qquad (2)$$

Substitute the value of *B* from equation (2) into equation (1).

$$\begin{aligned} G + 2 + G &= 26 \\ 2G + 2 &= 26 \\ 2G &= 26 - 2 \\ 2G &= 24 \\ G &= 12 \end{aligned}$$

Substitute this value for *G* into equation (2)

$$\begin{aligned} B &= G + 2 \\ B &= 12 + 2 \\ B &= 14 \end{aligned}$$

There are 12 girls and 14 boys living on Anklebiter Avenue.

Check: Since we have found the value for boys in equation (2) we need only check in equation (1).

Equation (1) LHS $= B + G$

$= 12 + 14$

$= 26$

$=$ RHS

5. Let Allan's income be A dollars and let Pei-shu's income be P dollars.

$A + P = 52\ 000 \quad (1)$

$A = P - 2\ 800 \quad (2)$

Substitute the value of A from equation (2) into (1).

$A + P = 52\ 000$

$P - 2\ 800 + P = 52\ 000$

$2P - 2\ 800 = 52\ 000$

$2P = 52\ 000 + 2\ 800$

$2P = 54\ 800$

$P = 27\ 400$

Substitute this value for P into equation (2)

$A = P - 2\ 800$

$A = 27\ 400 - 2\ 800$

$A = 27\ 400 - 2\ 800$

$A = 24\ 600$

Therefore Allan earns \$24 600 and Pei-shu earns \$27 400.

Check: Since we have found the value for Allan's income from equation (2) we need only check in equation (1).

Equation (1) LHS $= A + P$

$= 24\ 600 + 27\ 400$

$= 52\ 000$

$=$ RHS

6. Let x be the number of \$1 coins and let y be the number of \$2 coins.

   If we have x \$1 coins then we have $1x$ dollars.

   If we have y \$2 coins then we have $2y$ dollars.

$$x + 2y = 115 \qquad (1)$$

$$y = x + 14 \qquad (2)$$

   Substitute the value of y from equation (2) into (1).

$$\begin{aligned} x + 2y &= 115 \\ x + 2(x + 14) &= 115 \\ x + 2x + 28 &= 115 \\ 3x + 28 &= 115 \\ 3x &= 115 - 28 \\ 3x &= 87 \\ x &= 29 \end{aligned}$$

   Substitute this value for x into equation (2)

$$\begin{aligned} y &= x + 14 \\ y &= 29 + 14 \\ y &= 43 \end{aligned}$$

   Therefore there are 29 \$1 coins and 43 \$2 coins.

   Check: Since we have found the number of \$2 coins from equation (2) we need only check in equation (1).

Equation (1)

$$\begin{aligned} \text{LHS} &= x + 2y \\ &= 29 + 2 \times 43 \\ &= 29 + 86 \\ &= 115 \\ &= \text{RHS} \end{aligned}$$

7. (a) $4\boldsymbol{x} - \boldsymbol{y} = 6$ (1)

$2\boldsymbol{x} + 3\boldsymbol{y} = 17$ (2)

From equation (1) $\boldsymbol{y} = 4\boldsymbol{x} - 6$ (3)

Substitute this value for y into equation (2)

$$\begin{aligned} 2x + 3(4x - 6) &= 17 \\ 2x + 12x - 18 &= 17 \\ 14x &= 17 + 18 \\ 14x &= 35 \\ x &= \frac{35}{14} = \frac{5}{2} = 2\frac{1}{2} \end{aligned}$$

Substituting into equation (3)

$$\begin{aligned} y &= 4 \times \frac{5}{2} - 6 \\ y &= \frac{20}{2} - \frac{12}{2} = \frac{8}{2} = 4 \end{aligned}$$

Therefore the solution is $x = \frac{5}{2}$ and $y = 4$.

Check equation (1)

$$\begin{aligned} LHS &= 4x - y \\ &= 4 \times \frac{5}{2} - 4 \\ &= 6 \\ &= RHS \end{aligned}$$

Equation (2)

$$\begin{aligned} LHS &= 2x + 3y \\ &= 2 \times \frac{5}{2} + 3 \times 4 \\ &= \frac{10}{2} + 12 \\ &= 17 \\ &= RHS \end{aligned}$$

(b) $3\boldsymbol{x} + 4\boldsymbol{y} = 6$ (1)

$5\boldsymbol{x} + 3\boldsymbol{y} = 1$ (2)

From equation (1) $y = \dfrac{6 - 3x}{4}$

Substitute this value into equation (2)

$$\begin{aligned}
5x + 3\left(\frac{6-3x}{4}\right) &= 1 \\
4 \times 5x + 4 \times 3\left(\frac{6-3x}{4}\right) &= 1 \times 4 \\
20x + 3(6 - 3x) &= 4 \\
20x + 18 - 9x &= 4 \\
11x &= 4 - 18 \\
11x &= -14 \\
x &= \frac{-14}{11}
\end{aligned}$$

Substituting into equation (3)

$$y = \frac{6 - \left(3 \times \frac{-14}{11}\right)}{4}$$

$$y = \frac{6 + \frac{42}{11}}{4} = \frac{27}{11}$$

Therefore the solution is $x = \dfrac{-14}{11}$ and $y = \dfrac{27}{11}$

check equation (1)
$$\begin{aligned}
LHS &= 3x + 4y \\
&= 3 \times \frac{-14}{11} + 4 \times \frac{27}{11} \\
&= 6 \\
&= RHS
\end{aligned}$$

check equation (2)
$$\begin{aligned}
LHS &= 5x + 3y \\
&= 5x\frac{-14}{11} + 3 \times \frac{27}{11} \\
&= 1 \\
&= RHS
\end{aligned}$$

(c) Let ***m*** be the cost of a pair of men's shoes in dollars and ***l*** be the cost of a pair of ladies shoes in dollars.

50 ***m*** + 300 ***l*** = $47 500 (1)

61 ***m*** + 370 ***l*** = $58 510 (2)

From (1) $m = \frac{47500 - 300l}{50}$ (3)

Substitute this value for ***l*** into (2)

$$\begin{aligned}
61\left(\frac{47500 - 300l}{50}\right) + 370l &= 58510 \\
50 \times 61\left(\frac{47500 - 300l}{50}\right) + 50 \times 370l &= 50 \times 58510 \\
61(47500 - 300l) + 18500l &= 2925500 \\
2897500 - 18300l + 18500l &= 2925500 \\
2897500 + 200l &= 2925500 \\
200l &= 2925500 - 2897500 \\
200l &= 28000 \\
l &= \frac{28000}{200} = 140
\end{aligned}$$

Substituting into equation (3)

$$\begin{aligned}
m &= \frac{47500 - 300 \times 140}{50} \\
&= 110
\end{aligned}$$

Therefore men's shoes cost $110 and ladies shoes cost $140 check your solution. Since we have found the value for men's shoes from equation (2) we need only check in equation (1).

Equation (1)
$$\begin{aligned}
LHS &= 50m + 300l \\
&= 50 \times 110 + 300 \times 140 \\
&= 47500 \\
&= RHS
\end{aligned}$$

Solutions to a taste of things to come

1.

(a) $5[4(5m + 6) + 9] + d - 165$

where m represents the number of the month in which the person was born,

and d represents the number of the day on which the person was born.

(b) $5[4(5m + 6) + 9] + d - 165$

$= 5[20m + 24 + 9] + d - 165$

$= 100m + 120 + 45 + d - 165$

$= 100m + d + 120 + 45 - 165$

$= 100m + d$

2. (a) $C = 100 + 3n$ where C represents the daily costs,

and n represents the number of bracelets made.

(b) (i) $n = 120$

$C = 100 + 3n$

$C = 100 + 3 \times 120$

$C = 100 + 360$

$C = 460$

To make 120 bracelets in a day will cost $460.

(ii) $n = 800$

$C = 100 + 3n$

$C = 100 + 3 \times 800$

$C = 100 + 2\ 400$

$C = 2\ 500$

To make 800 bracelets in a day will cost $2 500.

(iii) $n = 1\ 000$

$C = 100 + 3n$

$C = 100 + 3 \times 1\ 000$

$C = 100 + 3\ 000$

$C = 3\ 100$

To make 1 000 bracelets in a day will cost $3 100.

(c)
$$\begin{aligned} C &= 100 + 3n \\ C - 100 &= 3n \\ \frac{C - 100}{3} &= n \\ n &= \frac{C - 100}{3} \end{aligned}$$

(d) If $C = 1\,600$ then
$$\begin{aligned} n &= \frac{C - 100}{3} \\ n &= \frac{1600 - 100}{3} \\ n &= \frac{1500}{3} \\ n &= 500 \end{aligned}$$

Therefore, 500 bracelets can be made for a cost of $1 600.

3. (a) 2.2 metres per second

= 2.2 × 60 metres per minute.

= 132 metres per minute.

(b) If a dive lasts 10 minutes, with 3 minutes spent on the bottom, there are seven minutes left to dive and ascend. That is, the penguin takes 3.5 minutes to reach the bottom.

(c) $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

By rearranging the formula, the depth or distance is equal to :

$$\begin{aligned} \text{distance} &= \text{speed} \times \text{time} \\ &= \frac{132 \text{ metres}}{\text{minute}} \times 3.5 \text{ minutes} \\ &= 462 \text{ metres} \end{aligned}$$

The penguin dives to 462 metres.

4. The two equations were:

$Q = 10P - 10$ (1) where Q represents the quantity supplied.

and P represents the price of each bowl in dollars.

and

$Q = -20P + 65$ (2) where Q represents the quantity demanded.

and P represents the price of each bowl in dollars.

By substituting equation (1) into equation (2) we will have one equation involving one variable.

$$
\begin{aligned}
10P - 10 &= -20P + 65 \\
10P - 10 + 20P &= -20P + 65 + 20P \\
30P - 10 &= 65 \\
30P - 10 + 10 &= 65 + 10 \\
30P &= 75 \\
\frac{30P}{30} &= \frac{75}{30} \\
P &= 2.5
\end{aligned}
$$

Substituting this value for P into equation (1) gives

$$
\begin{aligned}
Q &= 10P - 10 \\
Q &= 10 \times 2.5 - 10 \\
Q &= 25 - 10 \\
Q &= 15
\end{aligned}
$$

So, at a price of \$2.50, 15 bowls will be demanded and supplied.

Check:

Equation (1) $\quad \text{LHS} = Q = 15 \qquad \text{RHS} = 10P - 10 = 10 \times 2.5 - 10 = 25 - 10 = 15 = \text{LHS}$

$$
\begin{aligned}
\text{LHS} &= Q & \text{RHS} &= 10P - 10 \\
&= 15 & &= 10 \times 2.5 - 10 \\
& & &= 25 - 10 \\
& & &= 15 \\
& & &= \text{LHS}
\end{aligned}
$$

Equation (2)

$$
\begin{aligned}
\text{LHS} &= Q & \text{RHS} &= -20P + 65 \\
&= 15 & &= -20 \times 2.5 + 65 \\
& & &= -50 + 65 \\
& & &= 15 \\
& & &= \text{LHS}
\end{aligned}
$$

Solutions to post-test

1.

(a)

$$T = \frac{ab}{2}$$

$$2T = ab$$

$$\frac{2T}{b} = \frac{a\cancel{b}}{\cancel{b}}$$

$$\frac{2T}{b} = a$$

$$a = \frac{2T}{b}$$

(b) $y = \sqrt{3.1a} + 2.4$

$$y - 2.4 = \sqrt{3.1a}$$

$$(y - 2.4)^2 = (\sqrt{3.1a})^2$$

$$(y - 2.4)^2 = 3.1a$$

$$\frac{(y - 2.4)^2}{3.1} = \frac{\cancel{3.1}a}{\cancel{3.1}}$$

$$\frac{(y - 2.4)^2}{3.1} = a$$

$$a = \frac{(y - 2.4)^2}{3.1}$$

2.

(a)

$$4x + 3 = -2x - 9$$

$$4x + 3 + 2x = -2x - 9 + 2x$$

$$6x + 3 = -9$$

$$6x = -9 - 3$$

$$6x = -12$$

$$x = -2$$

Check: LHS $= 4x + 3$ RHS $= -2x - 9$

$= 4 \times -2 + 3$ $= -2 \times -2 - 9$

$= -8 + 3$ $= 4 - 9$

$= -5$ $= -5$

$= \text{LHS}$

(b)

$$x^2 - 9 = 0$$
$$x^2 = 9$$
$$x = \pm 3$$

Check: $x = 3$ LHS $= x^2 - 9$ $x = -3$ LHS $= x^2 - 9$

$= 3^2 - 9$ $= (-3)^2 - 9$

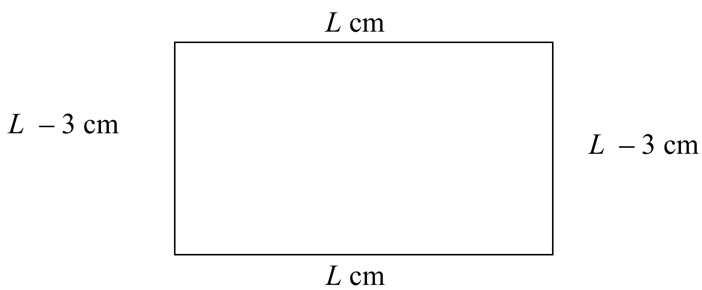
$= 9 - 9$ $= 9 - 9$

$= 0$ $= 0$

$= \text{RHS}$ $= \text{RHS}$

3. A diagram will help with the solution.

Let L cm represent the length of the rectangle, then $L - 3$ cm will be the width.



The perimeter is 42 and the equation to represent this is:

$$L + L - 3 + L + L - 3 = 42$$
$$4L - 6 = 42$$

$$4L = 48$$

$$L = 12$$

Since the length of the rectangle is 12 cm the width must be 9 cm.

Check: the perimeter is 9 + 12 + 9 + 12 = 42 which is what we would expect.

4. Using Pythagoras' Theorem

$$2^2 = (0.8)^2 + x^2$$

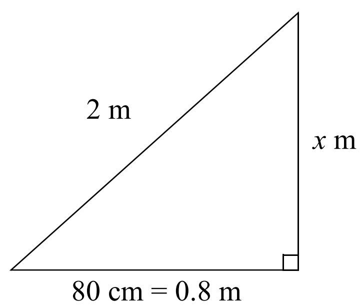
$$4 = 0.64 + x^2$$

$$x^2 = 4 - 0.64$$

$$x^2 = 3.36$$

$$x = \sqrt{3.36}$$

$$x \approx 1.83$$



The ladder reaches approximately 1.83 metres up the wall.

5. Let R represent the cost of a record in dollars and let T be the cost of a tape in dollars.

$$3R + 2T = 25.50 \qquad (1)$$

$$2R + 4T = 29 \qquad (2)$$

From (1) $2T = 25.50 - 3R$

$$T = \frac{25.50 - 3R}{2} \qquad (3)$$

Substitute this value for T into equation (2).

$$2R + 4T = 29$$

$$2R + 4\frac{(25.50 - 3R)}{2} = 29$$

$$2R + \frac{102 - 12R}{2} = 29$$

$$\frac{102 - 12R}{2} = 29 - 2R$$

$$102 - 12R = 2(29 - 2R)$$

$$102 - 12R = 58 - 4R$$

$$-8R = -44$$

$$R = 5.50$$

Substitute this value for R into equation (3)

$$T = \frac{25.50 - 3R}{2}$$

$$T = \frac{25.50 - 3 \times 5.50}{2}$$

$$T = \frac{25.50 - 16.5}{2}$$

$$T = \frac{9}{2}$$

$$T = 4.50$$

Therefore records cost \$5.50 and tapes cost \$4.50

Check:

Equation (1)

$$\begin{aligned} \text{LHS} &= 3R + 2T \\ &= 3 \times 5.50 + 2 \times 4.50 \\ &= 16.50 + 9 \\ &= 25.50 \\ &= \text{RHS} \end{aligned}$$

Equation (2)

$$\begin{aligned} \text{LHS} &= 2R + 4T \\ &= 2 \times 5.50 + 4 \times 4.50 \\ &= 11 + 18 \\ &= 29 \\ &= \text{RHS} \end{aligned}$$