On the impossibility of solitary Rossby waves in meridionally unbounded domains

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joint work with Dmitry Pelinovsky

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The initial hope

Quasi-geostrophic equation

$$\boxed{\frac{Dq}{Dt} = 0}$$

potential vorticity $\quad q = \nabla^2\psi + \beta y - F\psi \qquad (u, v) = (-\psi_y, \psi_x)$

β plane approximation $\quad f = f_0 + \beta y$

Froude number $\quad F = \dfrac{1}{L_r^2}$

Rossby radius of deformation $\quad L_R = \dfrac{\sqrt{g\bar{H}}}{f_0}$

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β plane approximation $f = f_0 + \beta y$

Froude number $F = \frac{1}{L_r^2}$

Rossby radius of deformation $L_R = \frac{\sqrt{g\bar{H}}}{f_0}$

Consider a meridional mean flow *U(y)* $\psi = -\int_0^y U(y)dy + \tilde{\psi}$

$$\boxed{\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\nabla^2\tilde{\psi} - F\tilde{\psi}\right) + (\beta + FU - U'')\frac{\partial\tilde{\psi}}{\partial x} + J\left(\tilde{\psi}, \nabla^2\tilde{\psi}\right) = 0}$$

$J(a, b) = a_x b_y - a_y b_x$

The initial hope

$$\boxed{\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\left(\nabla^2\tilde{\psi}-F\tilde{\psi}\right)+(\beta+FU-U'')\frac{\partial\tilde{\psi}}{\partial x}+J\left(\tilde{\psi},\nabla^2\tilde{\psi}\right)=0}$$

Linearize $\tilde{\psi}(x,y,t)=e^{ik(x-ct)}\phi(y)$

$$\boxed{(U-c)(\phi''-(F+k^2)\phi)+(\beta+FU-U'')\phi=0}$$

For constant mean flow $U(y)=\bar{U}$

$$c=-\frac{\beta}{F}+\frac{\beta+F\bar{U}}{F^2}(k^2+\ell^2)+\mathcal{O}((k^2+\ell^2)^2)$$

The initial hope

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(\nabla^2\tilde{\psi} - F\tilde{\psi}\right) + (\beta + FU - U'')\frac{\partial\tilde{\psi}}{\partial x} + J\left(\tilde{\psi}, \nabla^2\tilde{\psi}\right) = 0$$

Linearize $\tilde{\psi}(x, y, t) = e^{ik(x-ct)}\phi(y)$

$$(U - c)(\phi'' - (F + k^2)\phi) + (\beta + FU - U'')\phi = 0$$

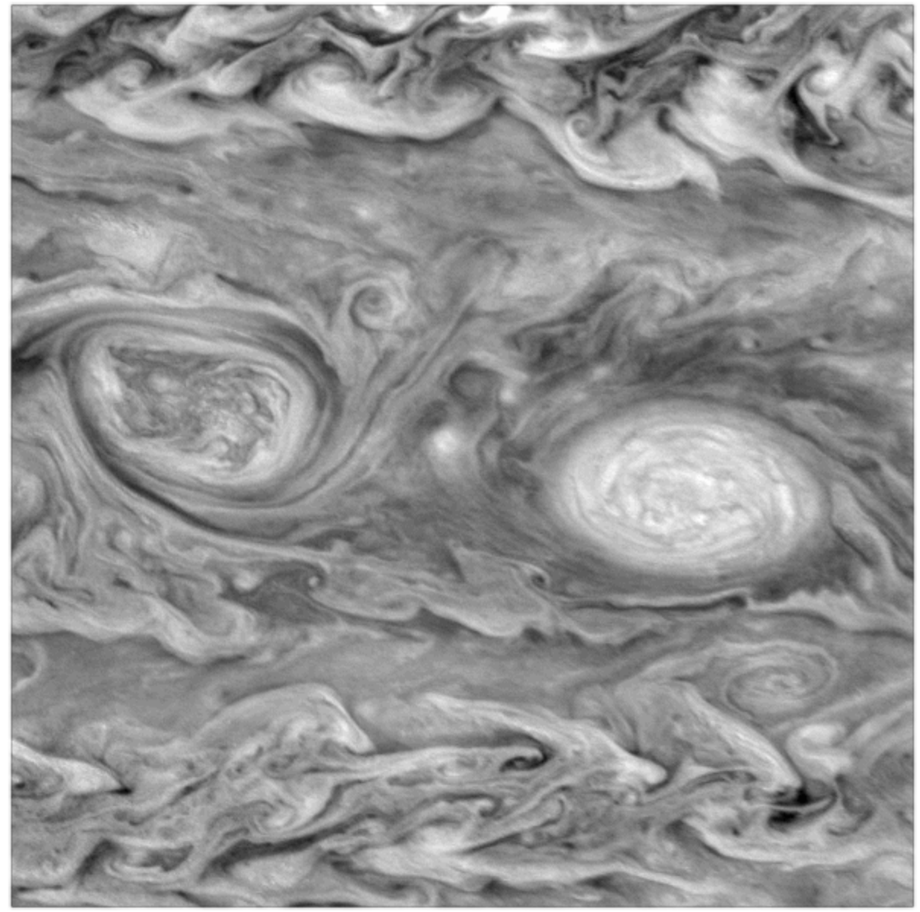
For constant mean flow $U(y) = \bar{U}$

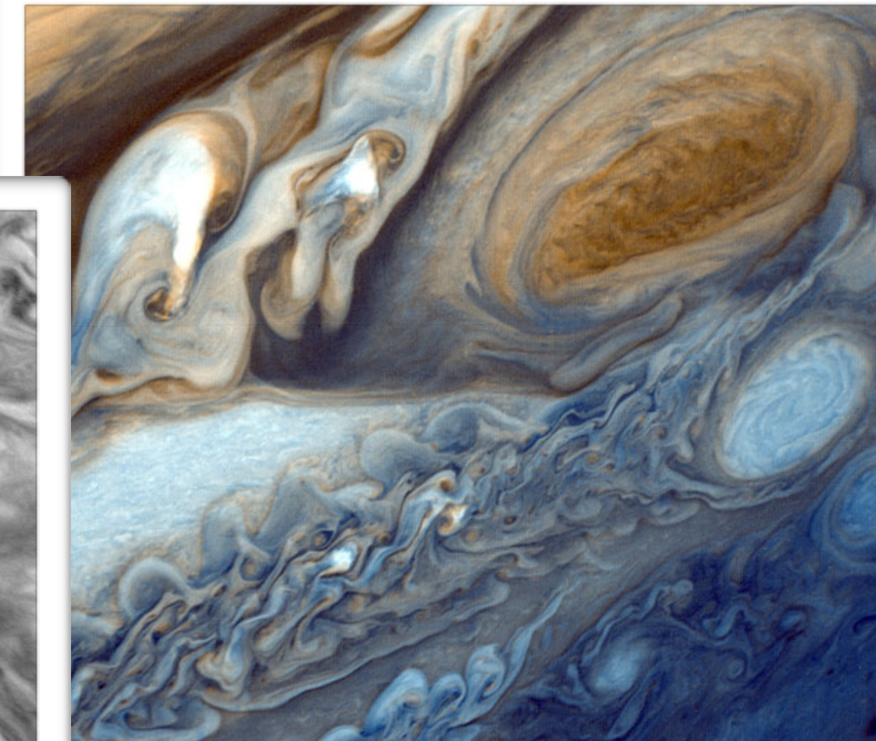
$$c = -\frac{\beta}{F} + \frac{\beta + F\bar{U}}{F^2}(k^2 + \ell^2) + \mathcal{O}((k^2 + \ell^2)^2)$$

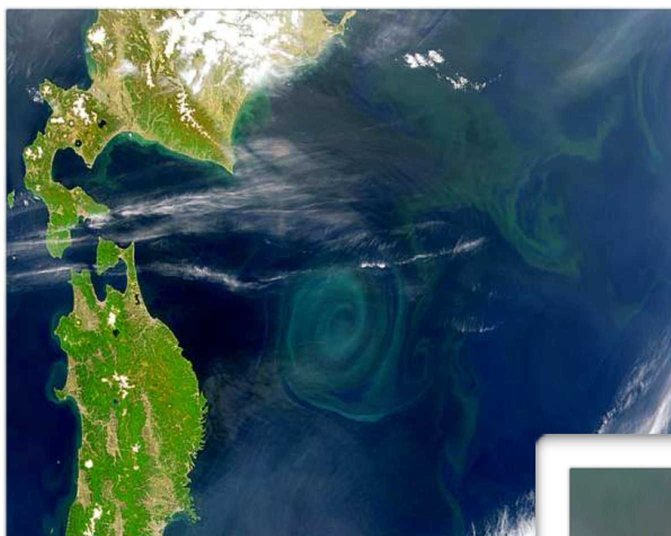
Zakharov-Kuznetsov equation

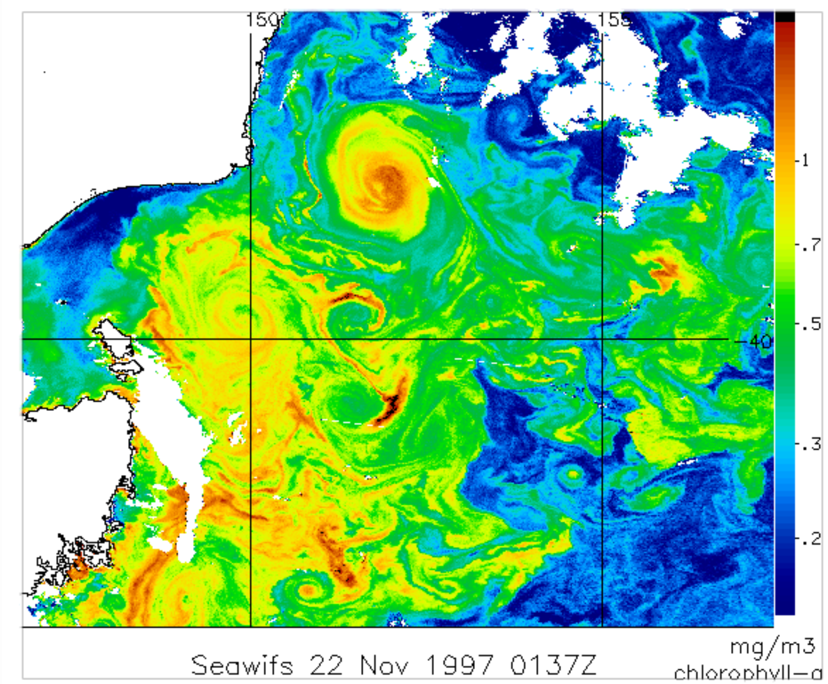
$$A_T = \lambda A_{XXX} + \zeta A_{XYY} + \mu A A_X$$

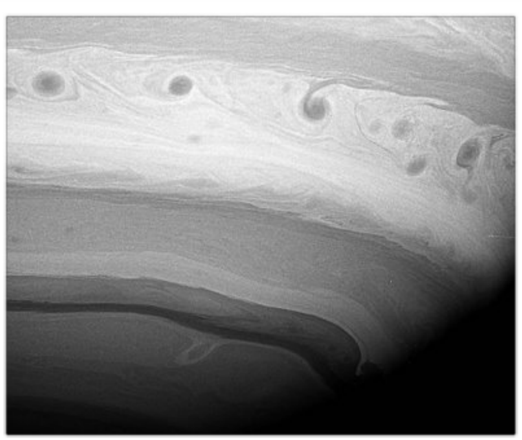
The initial hope





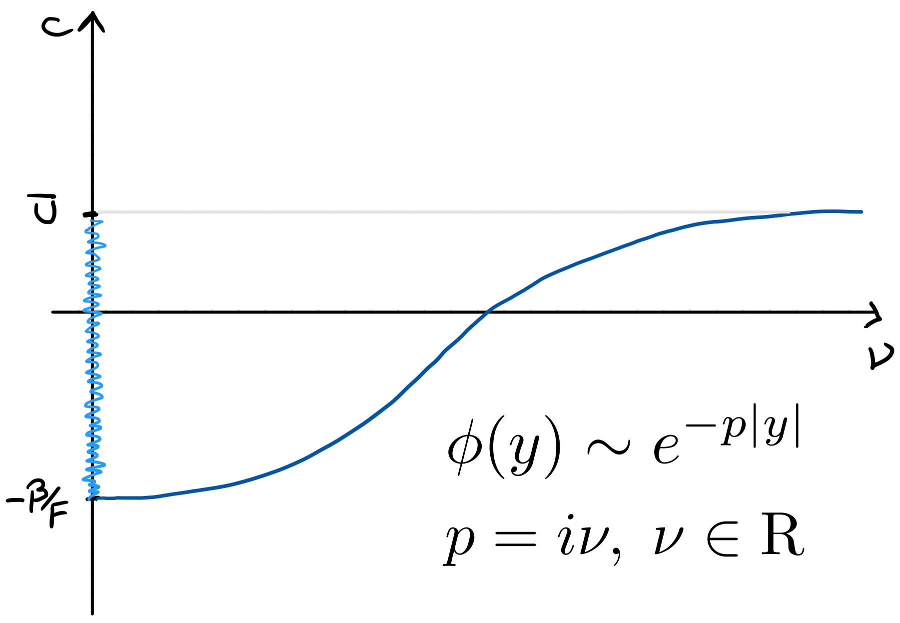






A more detailed look at the linear problem

$$(U-c)(\phi''-(F+k^2)\phi)+(\beta+FU-U'')\phi=0$$



$$\phi(y)\sim e^{-p|y|}$$

$$p=i\nu,\ \nu\in \mathrm{R}$$

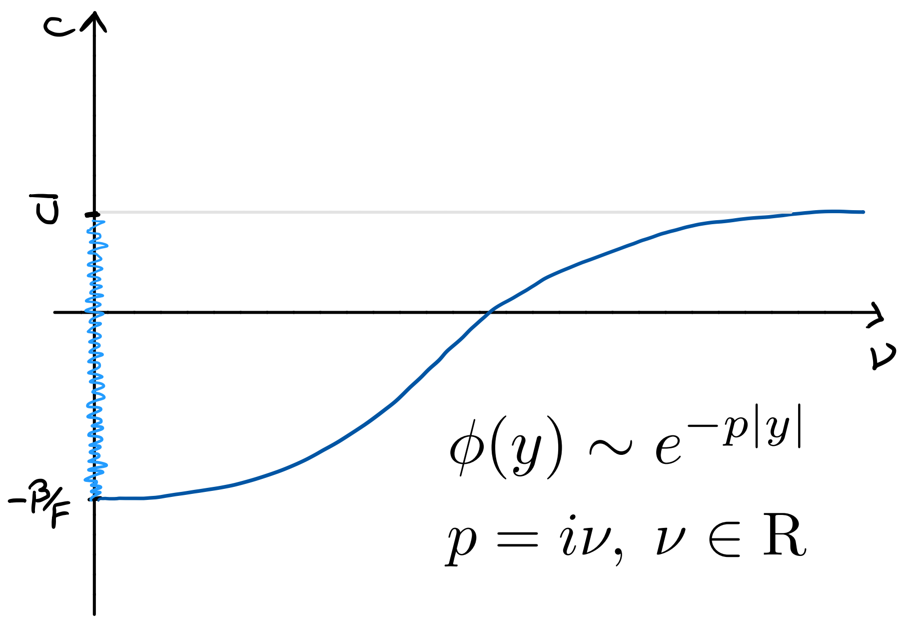
Consider $U(y)\to\bar{U}$ as $|y|\to\infty$

to seek localised

$$\phi(y)\to 0 \text{ as } |y|\to\infty$$

A more detailed look at the linear problem

$$) + (\beta + FU - U'')\phi = 0$$



nsider $U(y) \to \bar{U}$ as $|y| \to \infty$

seek localised

$$\phi(y) \to 0 \text{ as } |y| \to \infty$$

Here:

* $\bar{U} > 0$
* $c < -\dfrac{\beta}{F}$
* no critical layers: $U(y) + \dfrac{\beta}{F} > 0$ for all y

Consider $\mathcal{L}(c)\phi = 0$

$$\mathcal{L}(c) := (U - c)(\partial_y^2 - F) + \beta + FU - U''$$

unbounded, non-selfadjoint

Introducing $\varphi := (F - \partial_y^2)\phi$

$$\mathcal{M}\varphi = c\varphi$$

$$\mathcal{M} := U - (\beta + FU - U'')(F - \partial_y^2)^{-1}$$

bounded, non-selfadjoint

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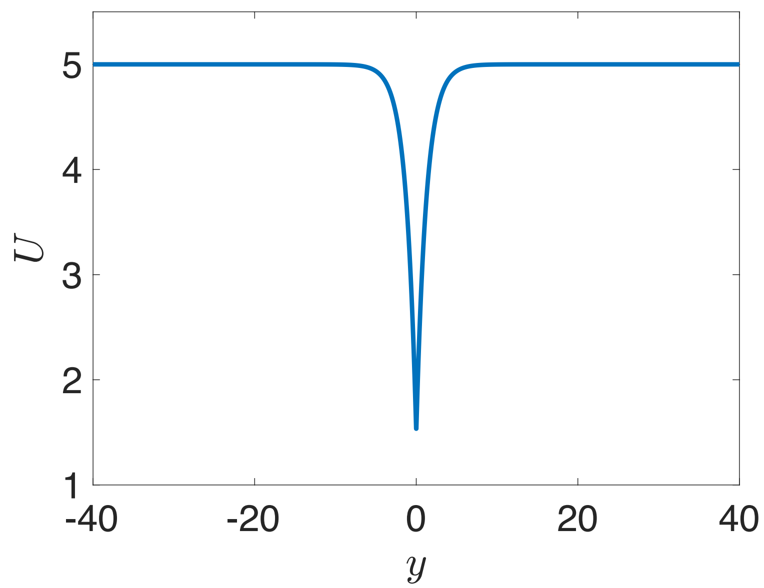
bounded, non-selfadjoint

We can prove

No eigenvalue c generally exist below $-\beta/F$ for smooth mean flow $U(y)$

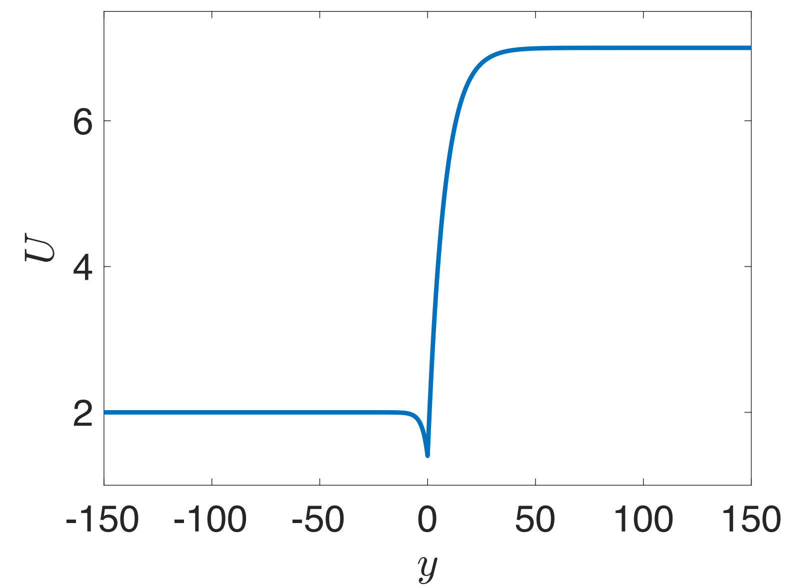
(e.g. for U bounded, and for convex $U''(y) \geq 0$)

Consider piecewise smooth mean flows



symmetric

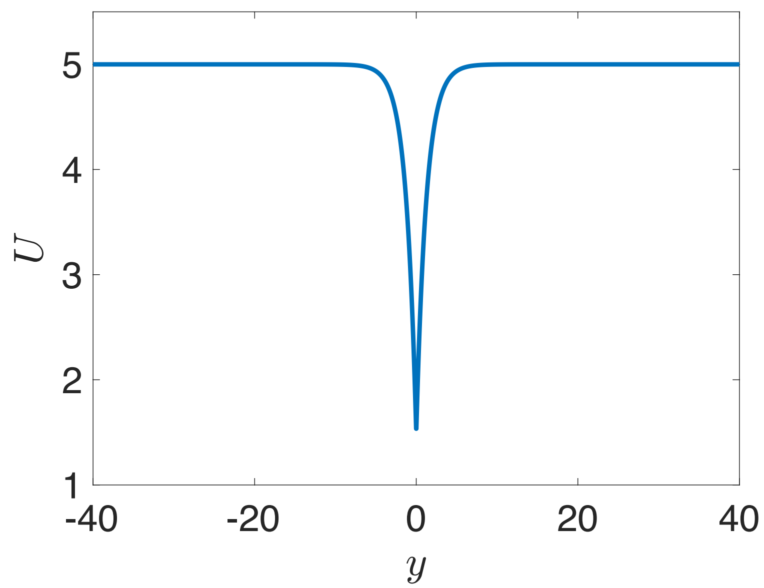
$$U(y) = \bar{U}\left[1 - a\exp(-b|y|)\right]$$



asymmetric

$$U(y) = \begin{cases} \bar{U}_-\left[1 - a_- \exp(b_- y)\right] & y < 0 \\ \bar{U}_+\left[1 - a_+ \exp(-b_+ y)\right] & y > 0 \end{cases}$$

Consider piecewise smooth mean flows



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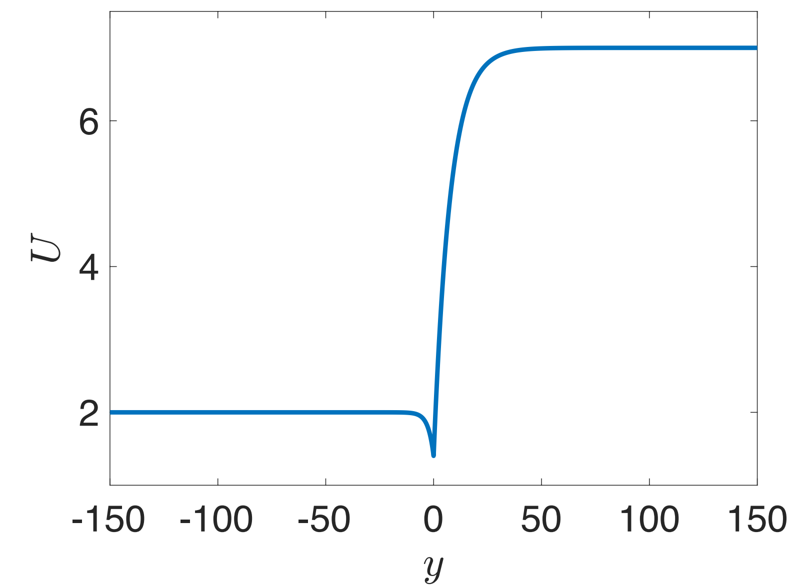

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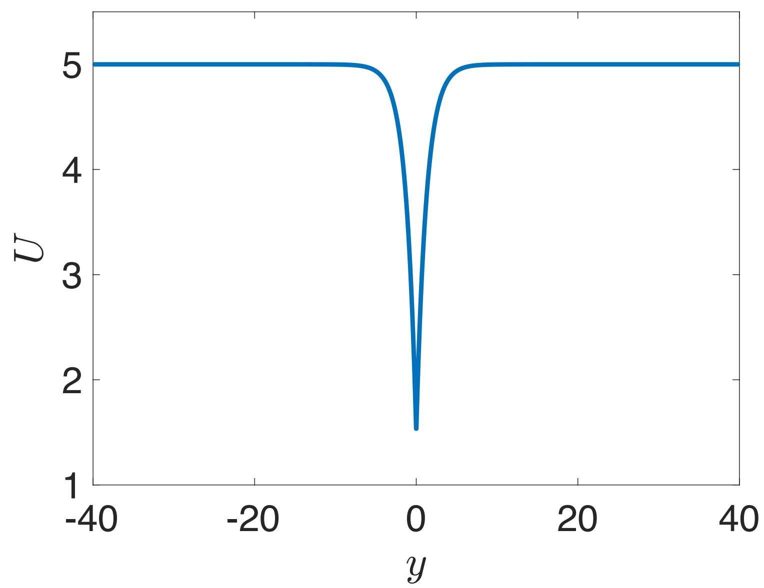

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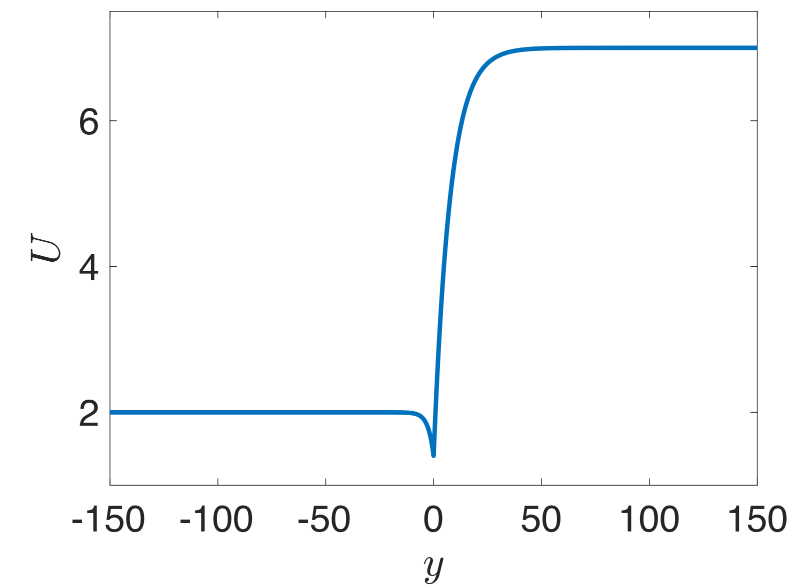

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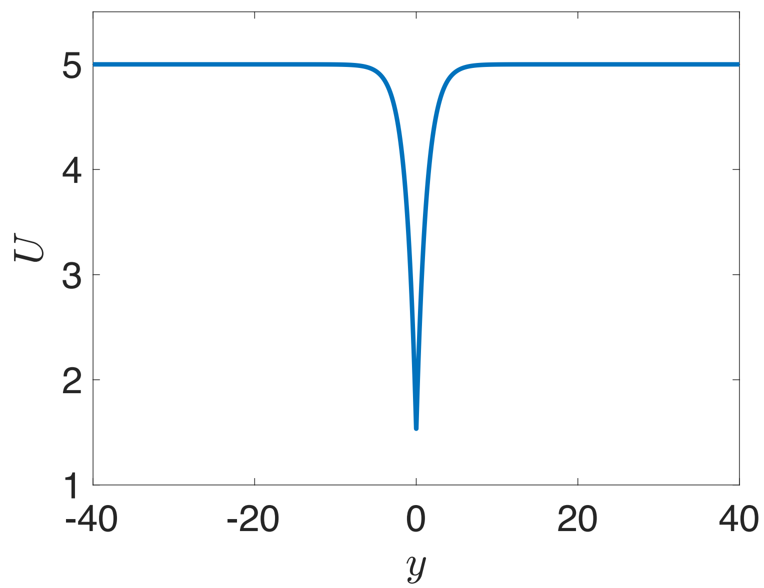
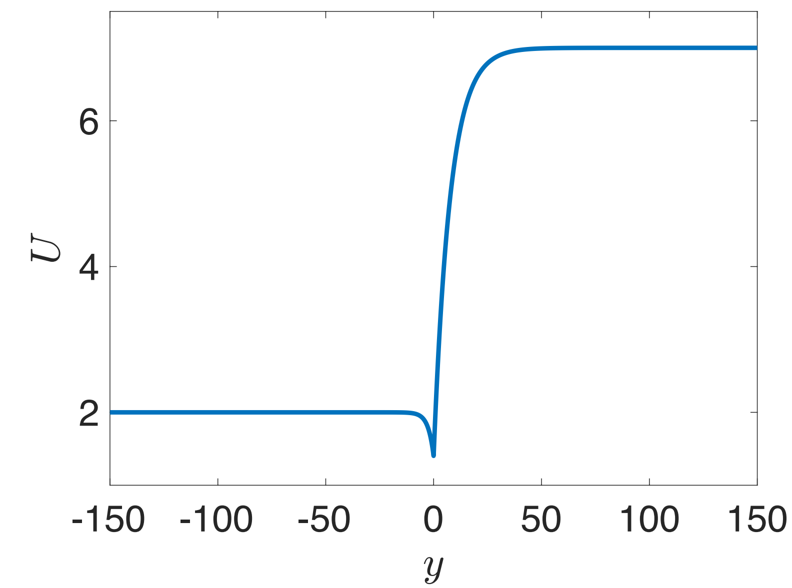
δ-function singularity

Consider piecewise smooth mean flows





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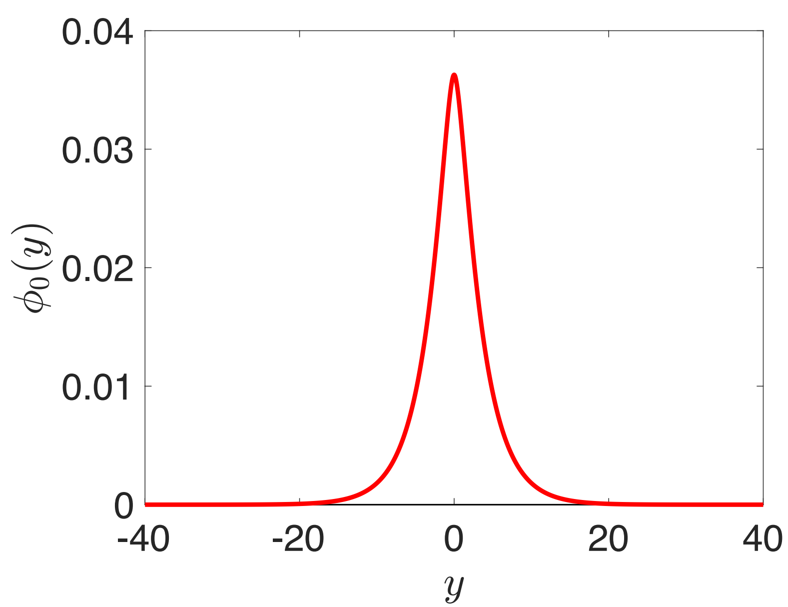

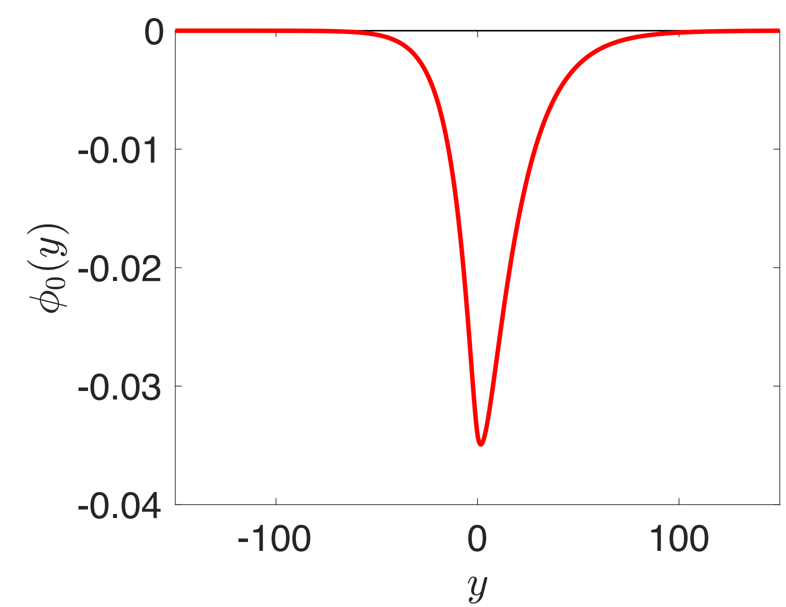

Derivation of solitary Rossby wave equations

Our aim/wish $$A_T = \mu A A_X + \lambda A_{XXX} + \zeta A_{XYY}$$

Zakharov-Kuznetsov equation

$$X = \epsilon(x - c_0 t), \qquad Y = \epsilon y, \qquad T = \epsilon^3 t$$

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Seek an asymptotic solution of the form

$$\tilde{\psi} = \epsilon^2\psi^{(2)} + \epsilon^3\psi^{(3)} + \epsilon^4\psi^{(4)} + \cdots$$

with leading-order perturbation stream function

$$\psi^{(2)} = A(\epsilon(x - c_0 t), \epsilon y, \epsilon^3 t)\,\phi_0(y)$$

where

$$\mathcal{O}(\epsilon^3): \qquad \mathcal{L}(c_0)\phi_0 = 0$$

$$\mathcal{O}(\epsilon^4): \qquad \mathcal{L}(c_0)\partial_X \psi^{(3)} = -2(U-c_0)A_{XY}\phi_0' \qquad \Longrightarrow \qquad \psi^{(3)} = -y\phi_0(y)A_Y(X,Y,T)$$

$$\mathcal{O}(\epsilon^5): \qquad \mathcal{L}(c_0)\partial_X \psi^{(4)} = \partial_T A(F-\partial_y^2)\phi_0 + 2(U-c_0)\,(y\phi_0)'\,A_{XYY}$$
$$- (A_{XXX}+A_{XYY})(U-c_0)\phi_0 - AA_X(\phi_0\phi_0''' - \phi_0'\phi_0'')$$

Fredholm alternative $\qquad \mathcal{L}^*(c)\theta = 0$

$$\Longrightarrow \qquad \boxed{A_T = \mu A A_X + \lambda A_{XXX} + \zeta A_{XYY}}$$

with parameters

$$\mu = \langle \theta_0, (\phi_0\phi_0''' - \phi_0'\phi_0'')\rangle$$
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$$\begin{aligned}\mu &= \langle\theta_0, (\phi_0\phi_0''' - \phi_0'\phi_0'')\rangle\\ \lambda &= \langle\theta_0, (U-c_0)\phi_0\rangle\\ \zeta &= \langle\theta_0, (U-c_0)(\phi_0 - 2\,(y\phi_0)') = 0\rangle\end{aligned}$$

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Can we get at least KdV?

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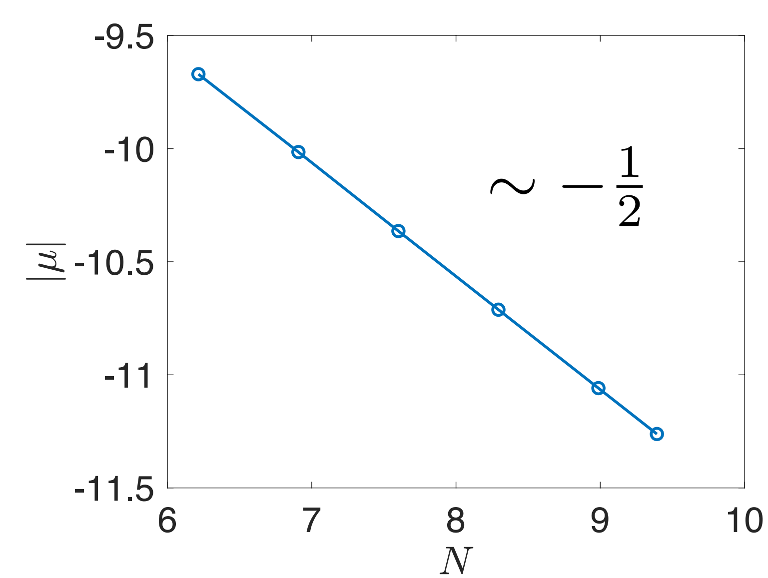
Can we get at least KdV?

For asymmetric mean flows only numerical evidence



We are not giving up yet: modified Zakharov-Kuznetsov equation

$$A_T = \kappa(AA_Y)_X + \nu A^2 A_X + \lambda A_{XXX} + \zeta A_{XYY}$$

$$X = \epsilon(x - c_0 t), \qquad Y = \epsilon y, \qquad T = \epsilon^3 t$$

Seek an asymptotic solution of the form

$$\tilde{\psi} = \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \epsilon^3 \psi^{(3)} + \cdots$$

with leading-order perturbation stream function

$$\psi^{(1)} = A(\epsilon(x - c_0 t), \epsilon y, \epsilon^3 t)\, \phi_0(y)$$

where

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$$\mathcal{O}(\epsilon^3): \qquad \mathcal{L}(c_0)\partial_X\psi^{(2)} = -2(U - c_0)A_{XY}\phi_0' - AA_X(\phi_0\phi_0''' - \phi_0'\phi_0'')$$

$$\psi^{(2)} = -y\phi_0(y)A_Y(X,Y,T) - \frac{1}{2}\phi_2(y)A(X,Y,T)^2$$

where $\mathcal{L}(c_0)\phi_2 = \phi_0\phi_0''' - \phi_0'\phi_0''$ with $\langle\phi_0, \phi_2\rangle = 0$

$\mathcal{O}(\epsilon^4)$:

$$\begin{aligned}\mathcal{L}(c_0)\partial_X\psi^{(3)} &= \partial_T A(F-\partial_y^2)\phi_0 + 2(U-c_0)\,(y\phi_0)'\,A_{XYY} - (A_{XXX}+A_{XYY})(U-c_0)\phi_0\\ &+ \partial_X(AA_Y)\,[2(U-c_0)\phi_2' + y\,(\phi_0\phi_0''' - \phi_0'\phi_0'')]\\ &+ \frac{1}{2}A^2A_X\,[\phi_0\phi_2''' + 2\phi_0'''\phi_2 - 2\phi_0'\phi_2'' - \phi_0''\phi_2']\end{aligned}$$

Fredholm alternative $\qquad \mathcal{L}^*(c)\theta = 0$

$$\boxed{A_T = \kappa(AA_Y)_X + \nu A^2 A_X + \lambda A_{XXX} + \zeta A_{XYY}}$$

with parameters

$$\kappa = -\langle\theta_0, [2(U-c_0)\phi_2' + y\,(\phi_0\phi_0''' - \phi_0'\phi_0'')]\rangle$$

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$\mathcal{O}(\epsilon^4)$:

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\mathcal{L}(c_0)\partial_X\psi^{(3)} &= \partial_T A(F-\partial_y^2)\phi_0 + 2(U-c_0)\,(y\phi_0)'\,A_{XYY} - (A_{XXX}+A_{XYY})(U-c_0)\phi_0 \\
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$$\mathcal{L}(c_0)\partial_X \psi^{(3)} = \partial_T A(F - \partial_y^2)\phi_0 + 2(U - c_0)\,(y\phi_0)'\, A_{XYY} - (A_{XXX} + A_{XYY})(U - c_0)\phi_0$$
$$+ \partial_X(AA_Y)\,[2(U - c_0)\phi_2' + y\,(\phi_0\phi_0''' - \phi_0'\phi_0'')]$$
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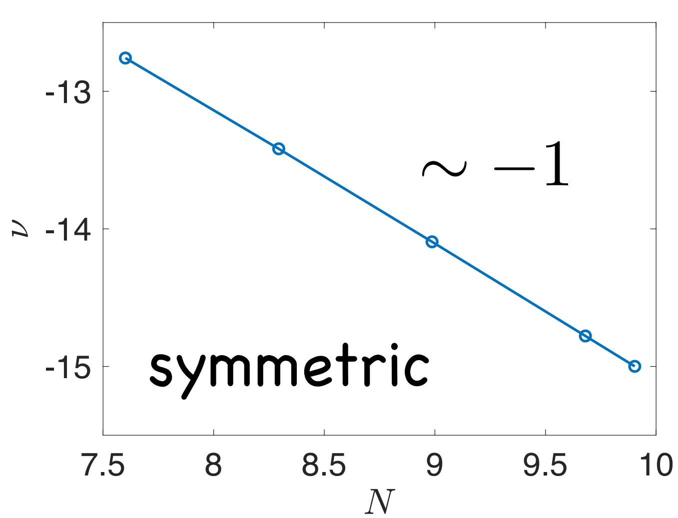
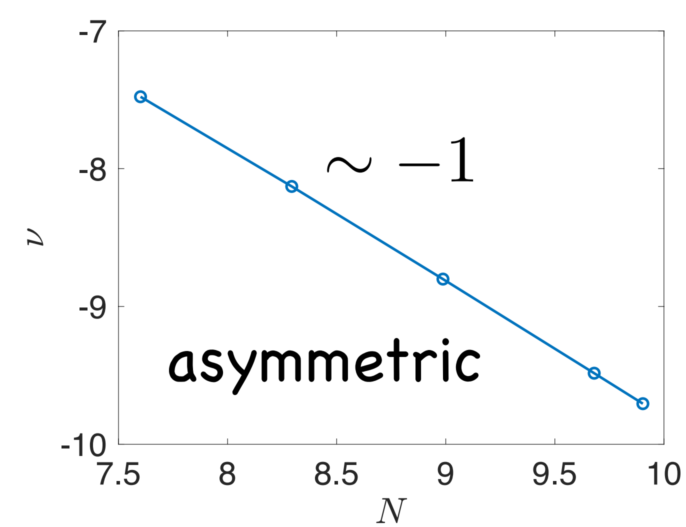
with parameters

$$\kappa = -\langle\theta_0, [2(U - c_0)\phi_2' + y\,(\phi_0\phi_0''' - \phi_0'\phi_0'')]\rangle = 0$$
$$\nu = -\frac{1}{2}\langle\theta_0, [\phi_0\phi_2''' + 2\phi_0'''\phi_2 - 2\phi_0'\phi_2'' - \phi_0''\phi_2']\rangle$$
$$\lambda = \langle\theta_0, (U - c_0)\phi_0\rangle$$
$$\zeta = \langle\theta_0, (U - c_0)(\phi_0 - 2\,(y\phi_0)') = 0\rangle$$

For v=0 only numerical evidence





In both cases the dynamics is effectively linear with zonal dispersion

$$A_T = \lambda A_{XXX}$$

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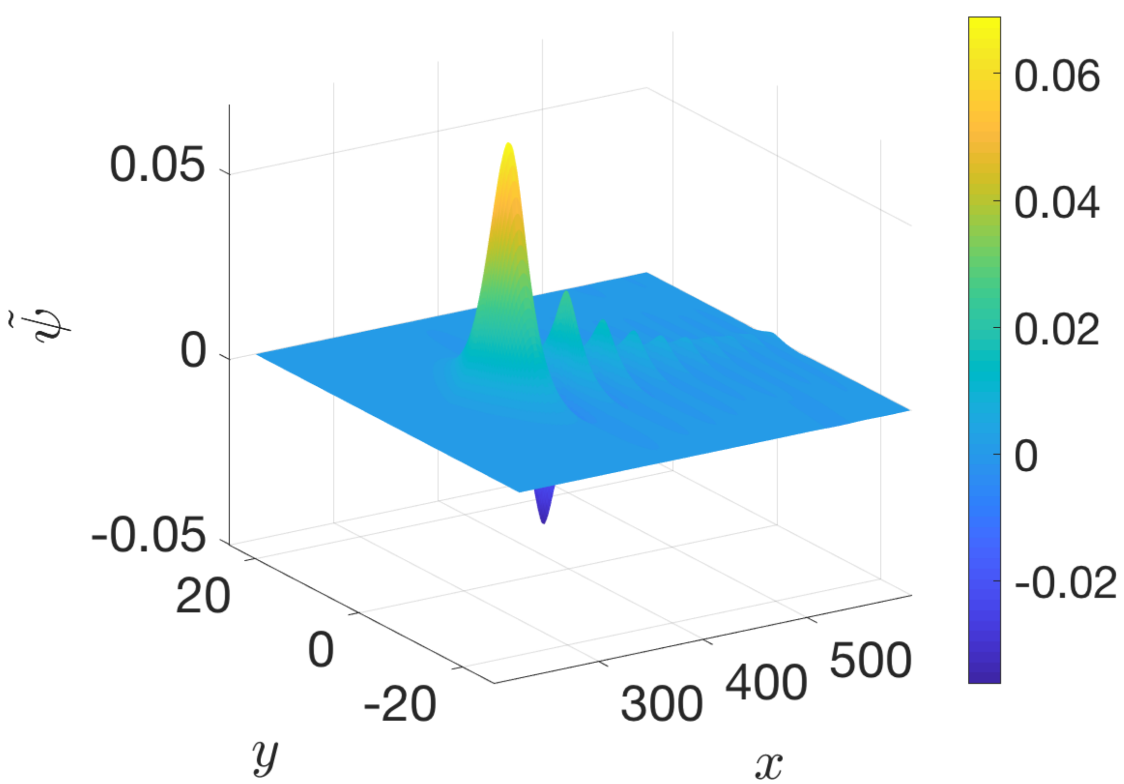
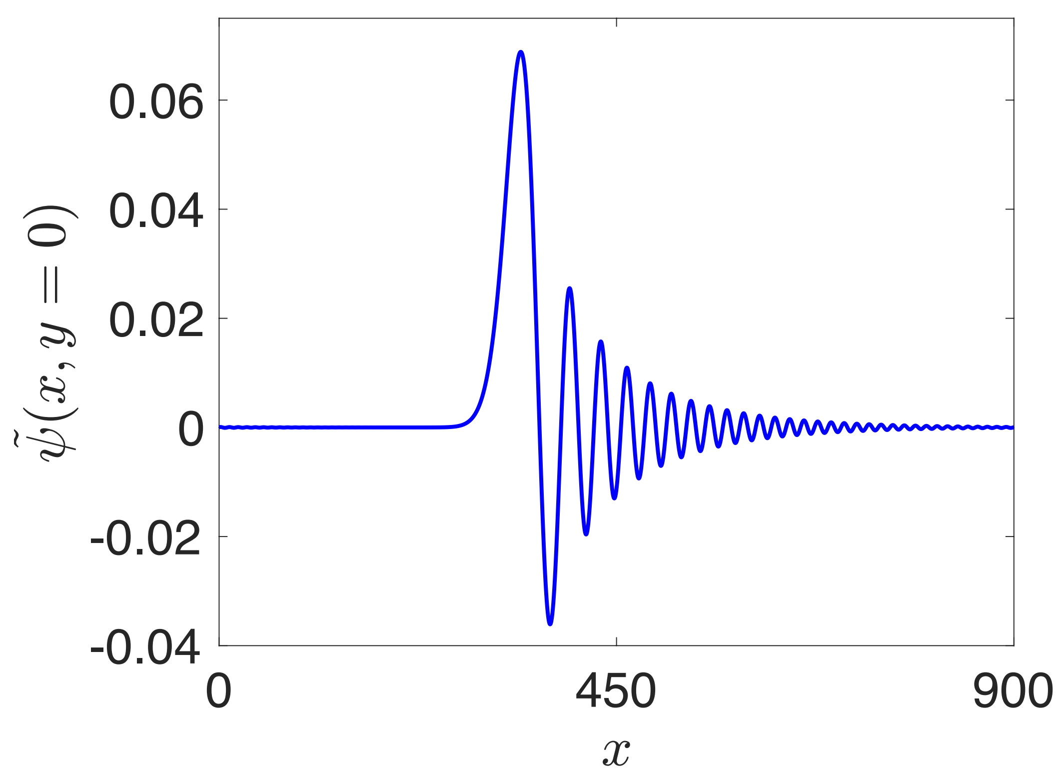
Numerical verification by considering evolution of localised initial conditions

$$\tilde{\psi}(x, y, t=0) = A_0 \,\mathrm{sech}^2(\mathrm{wx})\phi_0(\mathrm{y})$$

in full quasi-geostrophic equations





for symmetric mean flow

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Numerical verification by considering evolution of localised initial conditions

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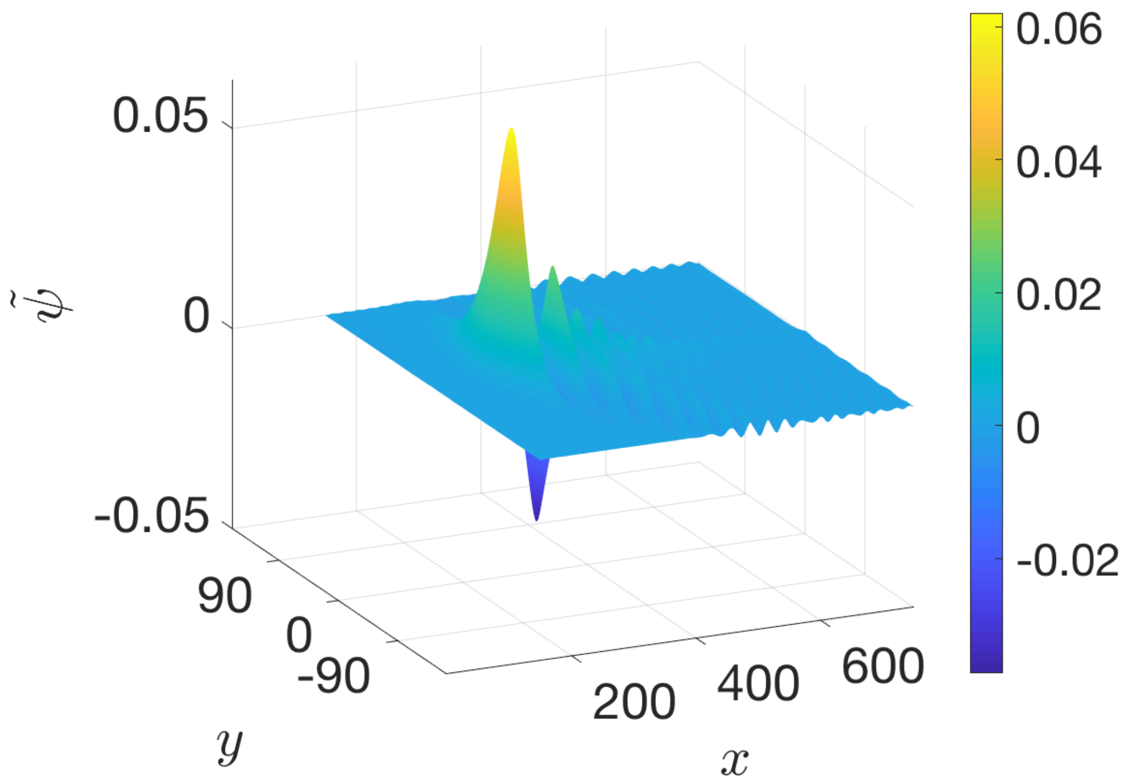

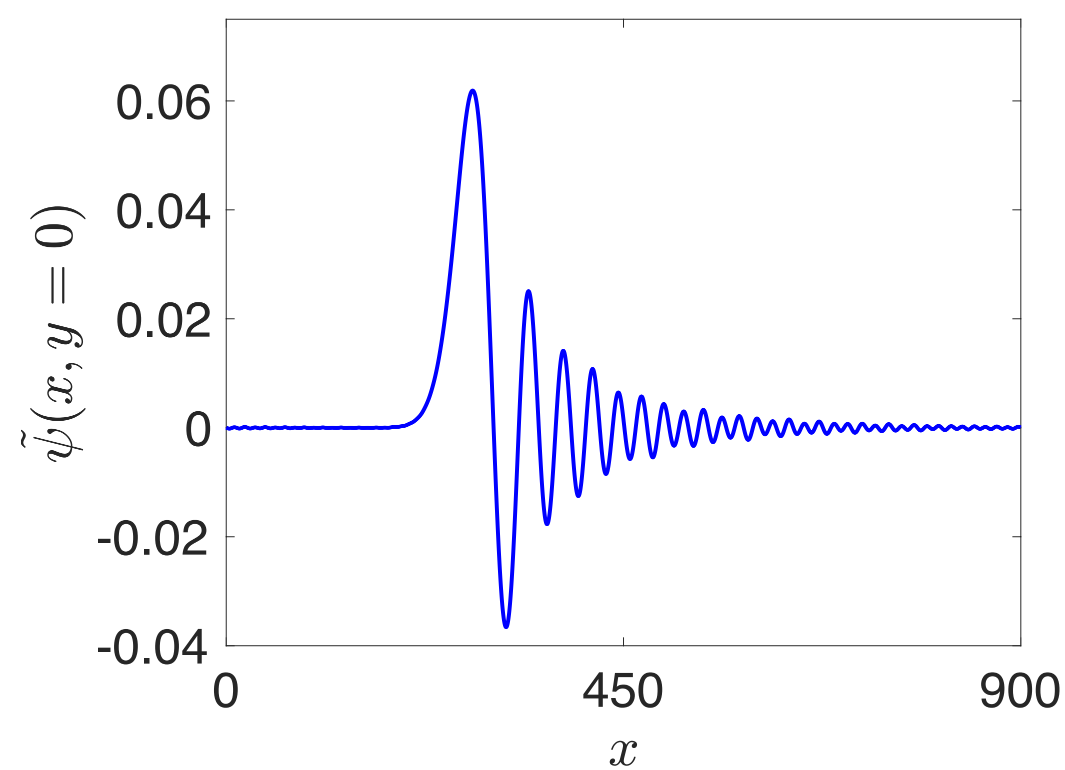

for asymmetric mean flow

Summary

- The dynamics of small localised large-scale perturbations in the quasi-geostrophic potential vorticity equation on unbounded domains is entirely linear
- with dispersion confined to the zonal direction

  Remark: KdV can be derived in meridionally confined domains

  Comment: issue of β-plane approximation in meridionally unbounded domain

Summary

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- with dispersion confined to the zonal direction

  Remark: KdV can be derived in meridionally confined domains

  Comment: issue of β-plane approximation in meridionally unbounded domain

Outlook

- what about unbounded domains with a single meridional boundary (e.g the antarctic circumpolar current)?

Thank you!

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Thank you, Roger!!!