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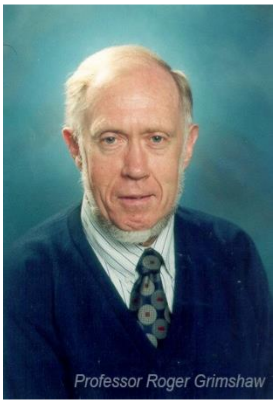
Workshop on Nonlinear Waves

USQ Toowoomba | 26 - 30 November 2018

Dedicated to the 80th birthday of Professor Roger Grimshaw, Fellow of the Australian Academy of Sciences, this five-day workshop aims to bring together world-leading experts on nonlinear waves in physical oceanography and beyond. The workshop will discuss contemporary research and trends in the nonlinear wave theory. It aims to promote interdisciplinary collaborations between Australian and New Zealand scientists with international researchers. Further, the workshop will place focus on advancing theoretical, numerical and experimental techniques across different branches of wave science, development of laboratory experiments, and cooperation for field experiments. Several popular public lectures will be given by famous researchers for a wide audience.


Professor Roger Grimshaw

Keynote Speakers

- Jerry Bona (University of Illinois at Chicago, USA)
- Roger Grimshaw (University College London, United Kingdom)
- Edward Johnson (University College London, United Kingdom)
- Karima Khusnutdinova (Loughborough University, United Kingdom)
- Boris Malomed (Tel Aviv University, Israel)
- Lev Ostrovsky (Colorado University, Boulder, USA)
- Efim Pelinovsky (Institute of Applied Physics, RAS, Nizhny Novgorod, Russia)
- Anthony Roberts (University of Adelaide, Australia)
- Victor Shrira (Keele University, United Kingdom)
- Vladimir Zakharov (Lebedev Physical Institute of Russian Academy of Sciences, Russia; University of Arizona, USA).

For further information

Website: **usq.edu.au/nonlinear-waves-workshop**

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Radar detection and characterization of nonlinear waves in the ocean

Stuart Anderson
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Abstract

Theoretical models of wave phenomena in the ocean environment play a central role in our attempts to address the challenges associated with climate change, renewable energy, deep ocean marine engineering and many other scientific, technological and geopolitical domains. While the sophistication of these models continues to grow, along with the computational capacity of the computers we use to implement them for numerical evaluation, there is still an important place for model validation in the form of *in situ* measurements using state-of-the-art technologies. Moreover, it is not infrequently the case that experiments reveal secondary phenomena that have been overlooked in the prevailing formulations of the primary topics of investigation.

Of the many modalities for remote sensing of ocean surface geometry and dynamics, those based on electromagnetic waves are preeminent on account of their non-invasive character, the scale, accuracy and precision of their measurements, access to signal dimensions of phase and polarization, achievable sampling rates and system dynamic range, diversity of candidate host platforms, span of intrinsic frequency bands (over 8 orders of magnitude), and amenability to bistatic and multistatic observation geometries. One might have thought that, given these credentials, a small industry would have evolved to apply radar to the problem of validating (or establishing the domains of validity) of models of subtle features of the theory, especially departures from linearity, stationarity, Gaussianity, and so on. Yet, with some notable exceptions, radar studies of the sea surface have tended to focus on refining our ability to determine those properties of the surface that are embodied in a linear formulation of the hydrodynamics.

This paper sets out to demonstrate, by a variety of examples, how radar can serve not only a means of quantitative assessment of nonlinear wave theories, but as a sensor able to exploit nonlinear properties to reveal phenomena that are not accessible when the echoes are interpreted through the lens of linear theory. Examples will be drawn from HF radar, microwave radar and optical passive radar.

Wave Breaking – Nonlinear Wave Phenomenon

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Wave breaking is one of the most significant dynamic processes at the ocean interface, and it influences the lower atmospheric boundary layer as well as the upper-ocean mixing. It facilitates or moderates the air-sea interactions including exchanges of energy, momentum, heat, gas, moisture, aerosol, and its understanding is important across the full range of ocean and coastal engineering applications, remote sensing of the ocean.

In the paper, mechanisms responsible for wave breaking will be reviewed and discussed, starting from academic one-dimensional laboratory tests to the continuous spectrum of two-dimensional wind-forced oceanic waves. It will be argued that modulational instability is the most likely reason behind the wave breaking in situ. Influences of wave directionality, wind forcing and extreme wind forcing will be highlighted.

Multidimensional solitons in dispersive complex media: structure and stability. Applications

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Abstract

This talk is devoted to a one of the most interesting and rapidly developing areas of modern nonlinear physics and mathematics – the analytical and advanced numerical study of the structure and stability of two- and three-dimensional solitons in dispersive complex media described by the generalized system which includes the Kadomtsev–Petviashvili and derivative nonlinear Schrodinger classes of equations and takes into account the generalizations relevant to various complex physical media, associated with the effects of high-order dispersion corrections, influence of dissipation and instabilities. This is consistent representation of the both early known and new original results obtained by author and also some generalizations in theory and numerical simulation of nonlinear waves and solitons in complex dispersive media. The analysis of stability of solutions is based on study of transformational properties of system Hamiltonian. The structure of possible multidimensional solutions is investigated using the methods of qualitative analysis of proper dynamical systems and analysis of solution asymptotics. Soliton interaction is studied numerically using specially developed numerical methods. Some applications in real physical media, such as soliton-like wave structures in plasma and fluids are discussed.

Water wave packet shoaling and dispersive shock waves

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Hydrodynamic dispersive shock waves (DSW) are well-known to occur in shallow water, for instance within the Korteweg–de Vries framework, and can describe the dynamics of undular or tidal bores, as reported by Zabusky and Kruskal (1965) as well as Trillo et al. (2016). DSW can be also observed in various nonlinear dispersive media, for instance in optics. Recently, Fatome et al. (2014) have experimentally confirmed that the propagation of optical pulses can lead to multiple optical dispersive shock events that interact as a result of four-wave mixing. We report an experimental study that has been conducted in a 200 m wave flume with a gradually varying bathymetry that consists of two flat parts connected by a 1/20 slope. We show that when breathers propagates over such type of bottom, DSW dynamics can be observed. The results are in very good agreement with the nonlinear Schrödinger equation in variable water depth.

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NON-LINEARITY IN TSUNAMI AND TIDAL BORES LINKED TO MULTIPHASE FLOW

by Hubert CHANSON, Xinqian (Sophia) LENG, Youkai LI

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Abstract: A tidal bore is an unsteady rapidly-varied open channel flow characterised by a rise in water surface elevation in estuarine zones, under spring tidal conditions. Related geophysical applications include the in-river tsunami bore and storm-surge induced bore. After formation, the bore is traditionally analysed as a hydraulic jump in translation and its leading edge is characterised by a breaking roller for $Fr_1 > 1.3$-1.5. The roller is a key flow feature characterised by intense turbulence and air bubble entrainment, associated with intense turbulent stresses across the water column and bed sediment motion.

A major challenge of many bore flows at geophysical scale is their three-phase nature that encompasses water, air and sediments. In this presentation, we will present new results in terms of bore roller turbulence and non-linearity linked to air bubble entrainment at the free-surface and sediment motion beneath the roller.

Keywords: Tidal bores, Tsunami, Bore roller, Air bubble entrainment, Sediment transport, Geophysical applications.

Computation of steady waves in shallow (and deep) water

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Gravity waves with a characteristic wavelength significantly larger than the mean water depth are often encountered in coastal engineering problems. Several accurate algorithms exist for computing (fully nonlinear) steady surface waves. However, none of these algorithms can compute long (cnoidal) waves, as they fail when the length-over-depth ratio exceed about $30 - -60$, depending on the method. Therefore, for long waves, one has to rely on shallow water approximations. However, these series are divergent and their accuracy is limited. Moreover, their calculation is not at all trivial and numerical difficulties appear for very long waves.

Here, we present an efficient algorithm for computing the steady solutions of the Euler equation. The method works efficiently for arbitrary depth (infinite depth as well as shallow water). With this algorithm, we show that solving the Euler equations is not more demanding than the resolution of simple models, such as KdV.

The forced coupled KdV equations as a model for internal waves in the atmosphere

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The coupled Korteweg–de Vries (KdV) equations model the resonant interaction of two modes of long, weakly nonlinear waves. Such behaviour has been proposed as the generation mechanism for the Morning Glory roll cloud over Cape York. Here we derive a forced version of the coupled KdV equations to model internal waves propagating on two layer interfaces with velocity shear and uneven bottom topography. The characteristics of these equations are discussed for two simple configurations of velocity shear and stratification. An algorithm is then derived to obtain steady, solitary wave solutions of these equations. Finally, the various types of solitary wave solutions and their stability is discussed.

Simple exact solutions of one-dimensional linear and nonlinear shallow water equations over sloping bottom.

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and
Moscow Institute of Physics and Technology

in cooperation with
A. Aksenov, K. Druzhkov and B. Tirozzi.

Abstract

First, we present a wide class of simple exact solutions to 2-D wave equation with constant velocity c describing the waves generated by special spatially localized sources. Far from the source and for large time t these solutions are localized near the circles (fronts) $|x|=ct$ and have the simple effective asymptotics. Then we take these asymptotics as initial data for the 1-D linear shallow water equations over sloping bottom and show that this type of initial data implies wide class of solutions of this equation in simple algebraic form. The application of the Carrier-Greenspan transform gives the class of exact solutions in the parametric form of nonlinear shallow water equations over sloping bottom. In particular, these solutions describe the interaction of "solitary waves" and "smooth steps" with the sloping bottom. We discuss applications of these solutions to the run-up problem, their relationship with Pelinovskii–Masova solutions and asymptotic generalization.

This work was supported by Russian Scientific Foundation, project No 16-11-10282.

Interaction of Korteweg–de Vries solitons with external sources

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We revise the solutions of the forced Korteweg–de Vries equation describing a resonant interaction of a solitary wave with external pulse-type perturbations. In contrast to previous works where only the limiting cases of a very narrow forcing in comparison with the initial soliton or a very narrow soliton in comparison with the width of external perturbation were studied, we consider here an arbitrary relationship between the widths of soliton and external perturbation of a relatively small amplitude. In many particular cases, exact solutions of the forced Korteweg–de Vries equation can be obtained for the specific forcings of arbitrary amplitude. We use the earlier developed asymptotic method by R. Grimshaw and E. Pelinovsky to derive an approximate set of equations up to the second-order on a small parameter characterising the amplitude of external force. The analysis of exact solutions of the derived equations is presented and illustrated graphically. It is shown that the theoretical outcomes obtained by asymptotic method are in a good agreement with the results of direct numerical modelling within the framework of forced Korteweg–de Vries equation.

On the impossibility of solitary Rossby waves in meridionally unbounded domains

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in cooperation with
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Abstract

Evolution of weakly nonlinear and slowly varying Rossby waves in planetary atmospheres and oceans is considered within the quasi-geostrophic equation on unbounded domains. When the mean flow profile has a jump in the ambient potential vorticity, localized eigenmodes are trapped by the mean flow with a non-resonant speed of propagation. We address amplitude equations for these modes. Whereas the linear problem is suggestive of a two-dimensional Zakharov–Kuznetsov equation, we found that the dynamics of Rossby waves is effectively linear and moreover confined to zonal waveguides of the mean flow. This eliminates even the ubiquitous Korteweg–de Vries equations as underlying models for spatially localized coherent structures in these geophysical flows.

Solitary wave trains and undular bores

Roger Grimshaw,

University College London, London, UK

In the weakly nonlinear long wave regime many physical systems, notably nonlinear internal waves in the coastal oceans, can be modelled with the variable-coefficient Korteweg-de Vries equation,

$$A_t + cA_x + \frac{cQ_x}{2Q}A + \mu AA_x + \lambda A_{xxx} = 0\,. \tag{1}$$

Here $A(x,t)$ is the amplitude of the relevant linear long wave mode, usually mode 1. c is the with linear long wave phase speed, Q is a linear magnification factor which ensures that wave action flux is conserved, μ, λ are system-dependent coefficients, and all these vary slowly with x. We show that an adaptation of Whitham modulation theory to a solitary wave train can be used to describe the evolution of undular bores in a variable medium.

Optimal shear instabilities of large-amplitude internal solitary waves

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P.-Y. Passaggia (Universite d'Orleans),
and B. L. White (UNC-Chapel Hill)

Abstract

The dynamics of perturbations to large-amplitude Internal Solitary Waves (ISW) with thin interfaces is analyzed by means of linear optimal transient growth methods. Optimal perturbations are computed through direct-adjoint loop iterations of the Navier–Stokes equations linearized around inviscid, steady ISWs obtained from the Dubreil-Jacotin–Long (DJL) equation. These disturbances are found to be localized wave-like packets that originate just upstream of the ISW self-induced zone of potentially unstable Richardson number, Ri < 0.25. They propagate through the base wave as coherent packets whose total energy gain increases rapidly with ISW amplitude. A local WKB approximation for spatially growing Kelvin–Helmholtz (KH) waves through the Ri < 0.25 zone captures properties (e.g., carrier frequency, wavenumber and energy gain) of the optimal disturbances except for an initial phase of non-normal growth due to the Orr mechanism. The non-normal growth can be a substantial portion of the total gain, especially for ISWs that are weakly unstable to KH waves. The linear evolution of Gaussian packets of linear free waves with the same carrier frequency as the optimal disturbances results in less energy gain than found for either the optimal perturbations or the WKB approximation due to non-normal absorption of disturbance energy in the leading face of the wave. Two-dimensional numerical calculations of the nonlinear evolution of optimal disturbance packets leads to the generation of large-amplitude KH billows that can emerge on the leading face of the wave that break down into turbulence in the lee of the wave. Observations of an unstable ISW (Moum et al., 2003 JPO 33, 2093–2112) are consistent with excitation by optimal disturbances.

Finite-amplitude compact wavepackets in rotating flows

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Steady solitary waves in non-rotating inviscid shallow water travel faster than any linear wave and so there is no mechanism through which they can lose energy. If however the undisturbed system is rotating as a whole, in solid body rotation about a vertical axis, then the resistance of the fluid to stretching along the axis of rotation adds an effective stiffness to the surface that becomes strongest for the longest waves. The phase velocity of linear waves thus increases without bound as their wavelength increases and so any localised steadily propagating disturbance generates a lee-wave wake, loses energy and disperses. Roger Grimshaw, working with J.-M. He and Lev Ostrovsky (1998), showed in the context of the Ostrovsky equation, that this leads to the decay of a KdV soliton in a time of order $1/(f^2\sqrt{A})$, where A is the amplitude and f is the Coriolis parameter. They presented numerical integrations demonstrating this, but also showing the soliton reappearing at longer times. Karl Helfrich (2007) showed, in the context of the Miyata–Choi–Camassa equation, that this apparent recurrence was because the initial disturbance had transformed into a finite-amplitude compact wavepacket with differing envelope and phase speeds. Roger and Karl (2008) then produced an analytical theory for these observations in the context of the Ostrovsky equation.

This talk will present joint work with Ashley Whitfield giving some more details of their theory, constructing the wavepackets numerically and presenting analytical forms for the weak and strong rotation limits, giving some observations on the initial value problem for the Gardner–Ostrovsky equation, and noting the form of the results in the case of anomalous dispersion (Alias, Grimshaw & Khusnutdinova 2014; Obregon & Stepanyants, 1998).

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Martingale solution to stochastic Korteweg - de Vries equation

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The celebrated Korteweg-de Vries equation (KdV for short), derived from the set of Eulerian shallow water and long wavelength equations, becomes a paradigm in the field of nonlinear partial differential equations. KdV appears as the lowest approximations of wave motion in several fields of physics.

A natural continuation of the study of the KdV equation seems to be a consideration of stochastic versions of such an equation. The KdV equation driven by random noise can be a model of several kinds of waves (e.g., surface water waves, waves in plasma) influenced by random factors. Two cases of the stochastic KdV equation are possible - the case with an additive noise and the case with a multiplicative noise.

We consider the stochastic Korteweg - de Vries equation with multiplicative random noise. We prove the existence of the martingale solution to the following stochastic KdV equation driven by noise of Lévy's type

$$(1)\quad \begin{cases} du(t,x)+\big(u_{3x}(t,x)+u(t,x)u_x(t,x)\big)dt=\int_Y F(t,u(t,x);y)\tilde{\eta}(dt,dy)+\Phi\left(t,u(t,x)\right)dW(t) \\ u(0,x)=u_0(x). \end{cases}$$

In the deterministic case, the assumption $u(t,x)=0$ for "large" $|x|$ leads to solitonic solutions, whereas the assumption in the periodic form $u(t,x)=u(t,x+l)$ leads to periodic solutions, so-called cnoidal waves, where l is the wavelength.

The stochastic KdV equation has been studied in several papers. To the best of our knowledge, there has been no result so far for the stochastic KdV equation driven by Lévy type noise. In the paper we extend the results of the existence of martingale solution to that case.

The presentation is based on the paper: Anna Karczewska and Maciej Szczeciński *Martingale solution to stochastic Korteweg - de Vries equation driven by Lévy noise.* arXiv:1708.03902.

On the Ostrovsky equation and related topics

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In this talk, I will overview some recent results related to the Ostrovsky equation [1]. Firstly, I will discuss the effects of the parallel shear flow on internal waves in a rotating ocean [2, 3]. We found first examples when the shear flow can change the sign of the rotation coefficient in the Ostrovsky equation, leading to unusual dynamics [3]. Secondly, I will discuss the dynamics of two distinct linear long wave modes with nearly coincident phase speeds, described by the system of coupled Ostrovsky equations. Interestingly, the dominant features of the complex dynamical behaviour observed in numerical simulations can be classified and interpreted in terms of the main features of the linear dispersion curves [3], resembling the qualitative theory of ODEs. Finally, I will briefly discuss how one can by-pass the so-called "zero-mass contradiction" in the class of periodic functions on a finite interval [4, 5], recovering the well-known scenario [6] in the limiting case of localised solutions on the "infinite" interval.

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The interaction of multi-lumps within the Kadomtsev–Petviashvili-1 equation: analytical and numerical results

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The interactions between the lumps and multi-lumps within the Kadomtsev–Petviashvili-1 (KP1) equation is studied both analytically and numerically. The dependence of the multi-lump structures on free parameters is discussed in details. Some interesting phenomena are obtained and demonstrated for the interactions of single lumps with each other and with more complex objects such as bi-lumps, as well as the interactions of bi-lumps with each other. Finally, the generation of these multi-lumps by a forced KP1 equation is discussed.

Generalised solitary gravity-capillary waves in the forced Korteweg-de Vries model

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Free surface flow past an applied pressure distribution in water of finite depth is presented in this work. Gravity and surface tension effects are included in the problem. Applying the asymptotic analysis, the resulting model is represented by the forced Korteweg-de Vries model (fKdV) where the Froude number and the Bond number are introduced to determine flow regime and capillary effects, respectively. At steady state, the fKdV model is solved numerically by the wavelet Galerkin method with Neumann boundaries in the far field. In the case of the capillary effect is dominated, a generalised solitary wave is found. This finding shows us more complete bifurcation diagram in the water wave theory.

Multidimensional solitons in optics and ultracold gases: Predictions and creation

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Abstract

It is commonly known that the interplay of linear and nonlinear effects gives rise to solitons, i.e., self-trapped localized structures, in a wide range of physical settings, including optics, Bose–Einstein condensates (BECs), hydrodynamics, plasmas, condensed-matter physics, etc. Solitons are considered as an interdisciplinary class of modes, which feature diverse internal structures. While most experimental realizations and theoretical models of solitons have been elaborated in one-dimensional (1D) settings, a challenging issue is prediction of stable solitons in 2D and 3D media. In particular, multidimensional solitons may carry an intrinsic topological structure in the form of vorticity. In addition to the “simple” vortex solitons, fascinating objects featuring complex structures, such as *hopfions*, i.e., vortex rings with internal twist, have been predicted too. A fundamental problem is propensity of multidimensional solitons to being unstable (naturally, solitons with a more sophisticated structure, such as vortical ones, are more vulnerable to instabilities). Recently, novel perspectives for the making of stable 2D and 3D solitons were brought to the attention of researchers in optics and BEC (such as the stabilization by means of the *spin-orbit coupling*, and the creation of *quantum droplets*). The present talk aims to provide an overview of the main results and ongoing developments in this vast field. An essential conclusion is the benefit offered by the exchange of concepts between different areas, such as optics, BEC, and hydrodynamics, to the generation of new results in physics, both theoretical and experimental.

Dispersive shock waves governed by the Whitham equation and their stability

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X. An and Noel F. Smyth
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Dispersive shock waves (DSWs), also termed undular bores in fluid mechanics, governed by the nonlocal Whitham equation are studied in order to investigate short wavelength effects that lead to peaked and cusped waves within the DSW. This is done by combining the weak nonlinearity of the Korteweg-de Vries equation with full linear dispersion relations. The dispersion relations considered are those for surface gravity waves, the intermediate long wave equation and a model dispersion relation introduced by Whitham to investigate the 120^o peaked Stokes wave of highest amplitude. A dispersive shock fitting method is used to find the leading (solitary wave) and trailing (linear wave) edges of the DSW. This method is found to produce results in excellent agreement with numerical solutions up until the lead solitary wave of the DSW reaches its highest amplitude. Numerical solutions show that the DSWs for the water wave and Whitham peaking kernels become modulationally unstable and evolve into multi-phase wavetrains after a critical amplitude which is just below the DSW of maximum amplitude.

Linear and nonlinear surface wave patterns for flow past a submerged source or doublet

Scott W. McCue

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This talk is concerned with steady free-surface flow past a submerged point source or doublet singularity. Behind the singularity, the wave pattern has a distinctive V-shape that is often characterised by the Kelvin wake angle. For the linearised regime, the problem is equivalent to flow past a submerged semi-infinite Rankine body with a rounded nose or submerged sphere. Interestingly, for both large and small Froude numbers, it turns out that the wake angle appears to be less than the Kelvin angle. For the nonlinear version of the problem, we apply a boundary integral-method based on Green's second formula. We are able to compute highly nonlinear solutions for which the waves have distinctive properties that are unlike their linear counterparts.

Nonlinear waves in rotating fluids

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Y. A. Stepanyants
University of Southern Queensland, Toowoomba, QLD, Australia

The non-trivial dynamics of nonlinear dispersive waves affected by the Coriolis force is discussed. Applications include surface and internal waves in the ocean, magnetic sound in plasma, and other phenomena. The corresponding model equation (rKdV equation) has the form

$$\left(u_t + c_0 u_x + \alpha u u_x + \beta u_{xxx}\right)_x = \gamma u,$$

where c_0 is the linear long wave velocity, α and β are, respectively, the nonlinearity and dispersion coefficients, and γ is proportional to the squared Coriolis frequency. This equation is not known to be integrable (except for the limits of $\gamma = 0$ or $\beta = 0$), and its solutions are defined by interplay of "two dispersions," the Boussinesq-type ($\sim\beta$) and Coriolis-type ($\sim\gamma$). Some specific features of this model found in different times are:

1. For the periodic and localized solutions, the mass integral is zero, $M \equiv \int u(x,t)\,dx = 0$.
2. In the case of "normal dispersion", when $\beta\gamma > 0$, there are no solitary waves on a constant background at all (the "antisoliton theorem").
3. In the long-wave case ($\beta = 0$) there exists a family of stationary periodic waves with a limiting wave consisting of a sequence of parabolic pieces.
4. An initial KdV soliton attenuates due to radiation of long small-amplitude waves and disappears as a whole entity in a finite time (the "terminal damping").
5. A long-time asymptotics of this solution can be a wave packet corresponding to the nonlinear Shrödinger equation.
6. A soliton can exist on a long-wave background, which compensates radiation losses.
7. Two solitons on such background reveal rather complex dynamics.
8. In the case of "anomalous dispersion" ($\beta\gamma < 0$) solitons do exist, as well as multisolitons and, possibly, chaotic dynamics.

All these issues will be briefly discussed in the presentation. Some data of laboratory experiments and oceanic modeling will be shown.

Long traveling waves in a fluid with a variable depth

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Abstract

Long wave propagation in a one- or two-layer fluid with a variable depth is studied for the specific bottom configurations, which allow waves to propagate over large distances. Such configurations are found within the linear shallow-water theory and determined by a family of solutions of the 2nd order ODE with three arbitrary constants. These solutions can be used to approximate the true bottom bathymetry. All such solutions represent smooth bottom profiles between two different singular points. The first singular point corresponds to the shore in non-stratified fluid or the point where the two-layer flow transforms into a uniform one. In the vicinity of this point the nonlinear shallow-water theory is used and the wave breaking criterion corresponding to the gradient catastrophe is found. The second bifurcation point corresponds to the infinite increase in water depth, which contradicts the shallow-water assumption. This point is eliminated by matching the "non-reflecting" bottom profile with a flat bottom. The wave transformation at the matching point is described by the second-order Fredholm equation, and its approximate solution is then obtained. The results extend the theory of surface and internal waves in inhomogeneous fluids demonstrating that waves can propagate over long distance in strongly inhomogeneous media.

Time-frequency analysis of nonlinear ship waves

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A spectrogram is a useful way of applying short-time discrete Fourier Transforms to visualise surface height measurements taken of ship waves in the real world. Previous research had identified a number of components of a spectrogram of a ship wave pattern, only two of which could be explained via linear theory. We use computer simulations of nonlinear ship waves to further study this problem. For this purpose, we consider nonlinear potential flow past a pressure applied to a free surface. Spectrogram analysis is applied to the computed nonlinear ship waves. Key features of the spectrograms, such as the linear dispersion curve, primary and secondary modes are identified. The multiple modes observed in this study bear a striking resemblance to components identified on spectrograms taken from previous experimental measurements of a high-speed ferry in the Gulf of Finland. If time permits, the effects of acceleration and finite-depth will be discussed.

Interactions of vector solitons in the model of a particle chain

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As has been shown in the paper [3], flexural transverse long waves of a small amplitude in an anharmonic chain of atoms can be described by the following vector equation:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - c^2 \frac{\partial^2 \mathbf{u}}{\partial x^2} - a \frac{\partial^2 \left(|\mathbf{u}|^2 \mathbf{u}\right)}{\partial x^2} - b_1 \frac{\partial^4 \mathbf{u}}{\partial x^4} - b_2 \frac{\partial^6 \mathbf{u}}{\partial x^6} = 0, \tag{1}$$

where $\mathbf{u} = \partial \overrightarrow{\xi}/\partial x$, and $\overrightarrow{\xi}$ is the chain displacement in the direction perpendicular to x-axis; c, a, b_1, and b_2 are real constant coefficients. Depending on the value of these coefficients, equation (1) can be reduced to several different evolution equations. In the typical case, wave propagation can be described by the vector modified Korteweg–de Vries (mKdV) equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \alpha |\mathbf{u}|^2 \frac{\partial \mathbf{u}}{\partial x} + \beta \frac{\partial^3 \mathbf{u}}{\partial x^2} = -\alpha_1 \mathbf{u} \frac{\partial |\mathbf{u}|^2}{\partial x}. \tag{2}$$

Here $\alpha_1 = \alpha$, but for the sake of flexibility we use different notations for the nonlinear coefficients, which allows us to consider in parallel another model with $\alpha_1 = 0$.

Equation (2) was derived in different physical contexts starting from 1970-th including description of plasma waves and waves in the chains of particles (see, e.g., Refs. [2, 1, 3, 4] and references therein). As was shown in Ref. [2], equation (2) is non-integrable in general, except the particular case when $\alpha_1 = 0$; in the latter case it becomes completely integrable.

In the papers cited above solitary and periodic stationary solutions were found within the framework of equation (2). Some of them represent helical periodic waves, others represent plane solitary waves which can propagate along the chain at different angles with respect to each other as shown in Fig. 1a). Each solitary wave performs a chain displacement in a certain plane only. Interaction of solitary waves is elastic in the itegrable case, when $\alpha_1 = 0$, but is non-elastic in general, when $\alpha_1 \neq 0$, except the case when they are in the same plane; in the latter case the basic equation (2) reduces to the completely integrable scalar mKdV equation.

In this work we present the results of numerical study of interactions between plane solitary waves of different polarization, i.e. having different orientation. We also consider a family of helical solitary waves (see

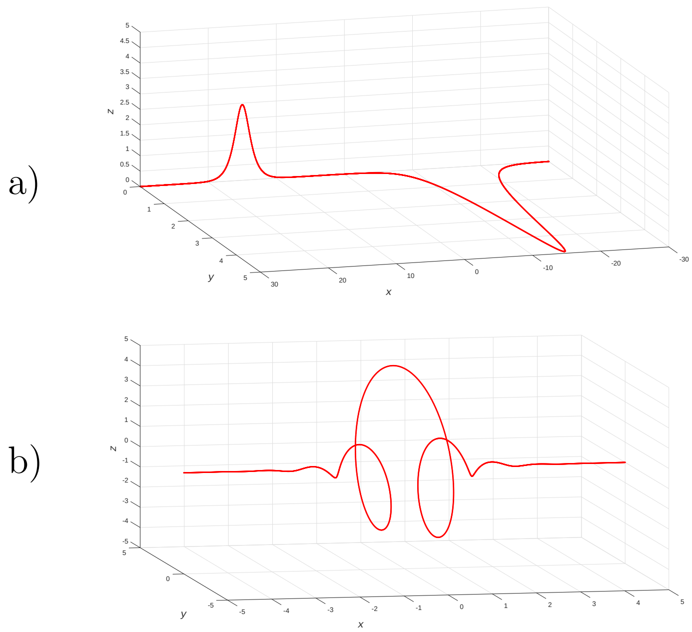

Figure 1: Examples of planar solitons of different (perpendicular) orientation (frame a) and a helical soliton (frame b) in Eq. (2).

Fig. 1b) and study interactions between them, as well as between the plane and helical solitary waves. In the case of interaction between helical solitary waves we show that the result depends upon the direction and strength of helicity. We present a comparison in the interactions of solitary waves in the integrable ($\alpha_1 = 0$), and non-integrable ($\alpha_1 = \alpha$) cases. The details of soliton interactions can be found in the website: https://eportfolio.usq.edu.au/view/view.php?t=dj2HQ3ioUZEOW0SfArbL

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Macroscale simulation of nonlinear waves via computation only on small staggered patches

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The multiscale gap-tooth scheme is built from given small microscale simulations of complicated physical processes to empower large macroscale simulations. By coupling small patches of simulations over un-simulated physical gaps, large savings in computational time are possible. Here we discuss generalising the gap-tooth scheme to the case of wave systems on staggered grids in both 1D and 2D. Classic macroscale interpolation provides a generic coupling between patches that achieves arbitrarily high order consistency between the emergent macroscale simulation and the underlying microscale dynamics. Eigen-analysis indicates that the resultant scheme empowers feasible computation of large macroscale simulations of wave systems even with complicated underlying physics. For examples, we simulate dam-breaking and turbulent floods by this scheme.

Exact soliton, periodic and superposition solutions to the fifth-order Korteweg–de Vries equation and derivation of a new equation for an uneven bottom

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Abstract

In the presentation we discuss several kinds of the exact solutions to the extended Korteweg–de Vries equation (name given by Marchant and Smyth [1]). This equation is obtained in a perturbation approach of second order with respect to small parameters, whereas KdV results from the same perturbation approach but limited to first order. That is why we call this equation KdV2 [2]. Alternatively, in the literature appears the name the fifth-order Korteweg–de Vries equation since it contains the fifth space derivative of the unknown function.

In 2014 we derived the exact soliton solution to KdV2 assuming it in the form $A\,\mathsf{sech}^2[B(x-vt)]$, that is, in the same form as the KdV solution but with different coefficients. The success of this result led us to the conjecture that the other kinds of exact KdV solutions could exist in the same functional form for KdV2. In [3] we derived the exact periodic (cnoidal) solutions in the form $A\,\mathsf{cn}^2[B(x-vt),m]+D$. In [4] we found the exact periodic solutions to KdV2 (named superposition solutions by Khare and Saxena [5]) in the forms $\frac{A}{2}\{\mathsf{dn}^2[B(x-vt)\pm\mathsf{cn}[B(x-vt)\mathsf{dn}[B(x-vt)\}+D$.

The KdV2 equation supplies one more condition on the coefficients of the solution than KdV, therefore the ranges of coefficients of solutions to KdV2 are usually narrower than those of KdV. Nevertheless, KdV2 admits, besides "normal" cnoidal solutions, the "inverted" cnoidal waves, too.

Additionally, we present the new second order equation for the case of an uneven bottom of the arbitrary shape. This equation is derived under assumption that small parameters $\alpha=O(\beta)$ and $\delta=O(\beta^2)$. The corresponding equation for the case $\alpha=O(\beta)$ and $\delta=O(\beta)$ cannot be obtained correctly since in this case the Boussinesq equations cannot be made compatible.

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Blow-up of vorticity waves in homogeneous and weakly stratified boundary layers

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Abstract

High Reynolds number boundary layers are ubiquitous in nature and engineering context. Often the velocity shear coexists with density stratification. For example, in the ocean the boundary layer in the water adjacent air-water interface plays a key role in ocean atmosphere interaction; the first 2.5 m have the heat capacity of the entire atmosphere above. This boundary layer might be stratified with a stable stratification caused by solar heating or by entrainment of air bubbles produced by breaking waves. The oceanic bottom boundary layers are often stratified due to sediment entrainment. Engineering provides ennumerous variety of laminar and turbulent boundary layers, which are often unstable. In the linear setting the nonstratified boundary layers support a single vorticity mode which might be stable or unstable, while in the presence of stratification there appears also an infinite number of modes corresponding to internal gravity waves. Here we consider nonlinear evolution of the vorticity mode (with or without weak density stratification) in the boundary layers in semi-infinite fluid with either the "rigid lid" or " no-slip" boundary condition.

The lecture provides both a brief overview of the activity in this area and a thorough asymptotic derivation of a novel nonlinear evolution equation for the vorticity mode. Independently of the orientation of the wavevector of a small amplitude perturbation to leading order vorticity mode motions are just a perturbation of the longitudinal velocity, while the

transverse and vertical velocity components are much smaller. This key property is preserved in weakly stratified flow, which enables us to get a single closed evolution equation for the motions with comparable scales in both horizontal directions. The derived model represents a "distinguished limit": it simultaneously takes into account nonlinearity, dissipation, dispersion due to shear and dispersion caused by weak stratification as well. The derived evolution equation for the amplitude A of longwave perturbations of boundary layer reads,

$$\partial_x \left\{ A_\tau - \alpha_1 A A_x - \beta_1 \hat{K}[A_x] - \gamma A \right\} = \beta_2 \Delta_h A$$

where x is the coordinate in the direction of the flow, y is the transverse horizintal coordinate, τ is slow time, the coefficients α_1, β_1 are determined by the parameters of the shear in the boundary layer, γ depends on the Reynolds number and shear profile, while β_2 is determined by both the stratification and shear profiles. The dispersion operators $\hat{K}$ and Δ_h are two-dimensional:

$$\hat{K}[\varphi(\mathbf{k})] = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |\mathbf{k}| \varphi(\mathbf{r}_1) e^{(i\mathbf{k}(\mathbf{r}-\mathbf{r}_1))} d\mathbf{k} d\mathbf{r}_1, \quad \Delta_h = \partial_x^2 + \partial_y^2,$$

and $k = |\mathbf{k}|$ and $k^2 = k_x^2 + k_y^2$.

The vertical localisation of the motions is prescribed by the eigenfunctions of the boundary value problem (the Taylor-Goldstein-Orr-Sommerfeld equation with weak stratification, weak viscosity in the range of long but finite wavelengths with the standard boundary conditions).

For the reduced version of this equation without explicit dissipation ($\gamma = 0$) and stratification ($\beta_2 = 0$), the existence of blow-up was first shown analytically by Dyachenko and Kuznetsov (1994), here we investigate the full equation numerically. Localised initial conditions with amplitude exceeding a threshold specified by the parameters of the flow and the shape of initial perturbation lead to blow-up, which implies dramatic intensification of mixing. The account of dissipation and stratification extends the area in the parameter space where the blow-up is impossible by raising the threshold. Crucially, it does not shrink the domain with blow-up to zero.

Scattering of surface waves by a periodic system of spikes at the bottom.

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The reflection of small-amplitude surface waves from a sharp change of bottom morphology is analysed. A 2D comb-like surface is used as a model of ‘spiky’ bottom morphology. This surface can be characterised by two parameters, viz., height of the spikes and period of the comb, which can have arbitrary relative values. The problem is treated analytically by means of conformal mapping. The amplitude of transmitted and reflected waves is explicitly calculated in terms of the geometrical parameters of the comb-like surface. It is shown that the geometrical parameters of of bottom morphology can be combined in one aggregated parameter (‘effective’ height) that can be evaluated by taking far-field limit of the solution in transformed coordinates [1].

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Nematic liquid crystals, resonant undular bores and the fifth order Korteweg–de Vries equation

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Undular bores are an unsteady waveform which is widely observed in nature, familiar examples being tidal bores and tsunamis. Standard undular bores are a continually evolving and spreading waveform consisting of solitary waves at one edge and linear dispersive waves at the other. While nonlinear, dispersive wave equations, such as the Korteweg-de Vries (KdV) and nonlinear Schrödinger (NLS) equations, are well known and studied for their solitary wave solutions, these equations also possess undular bore solutions, also termed dispersive shock wave solutions. Undular bore solutions are typically found as simple wave solutions of the Whitham modulation equations of the governing equation when these equations are hyperbolic and so the underlying steady periodic wave solution is stable. The undular bore solution is thus found as a modulated periodic wave. This talk will look at non-standard undular bores which are not of this standard form. These non-standard bores are resonant undular bores. For such bores, linear dispersive waves are in resonance with the component waves of a bore. This resonant wavetrain has a dominant effect on the bore and its structure, which greatly differs from a standard bore. A resonant undular bore is studied through the example of the KdV equation with a fifth derivative. This equation arises for internal waves under an ice sheet and as an approximation for nonlinear optical beams in liquid crystals. It is found that new solutions based on Whitham modulation theory can describe resonant undular bores.

The effects of interplay between the rotation and shoaling for a solitary wave on variable topography

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The specific features of solitary wave dynamics within the framework of the Ostrovsky equation with variable coefficients are considered in relation to surface and internal waves in a rotating ocean with a variable bottom topography. It is shown that for solitary waves moving toward the beach, the terminal decay caused by the rotation effect can be suppressed by the shoaling effect. Two basic examples of a bottom profile are analyzed in detail and supported by direct numerical modelling. One of them is a constant-slope bottom and the other is a specific bottom profile providing a constant amplitude solitary wave. Estimates with real oceanic parameters show that the predicted effects of stable soliton dynamics in a coastal zone can occur, in particular, for internal waves.

Breathers in the stratified fluid: generation, dynamics and stability

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Existence of internal wave breathers in a density stratified fluid has been predicted by the asymptotic theory within the framework of KdV-type equations. Observations of long-wave internal breather-like waves off the coast of New Jersey in the Yellow Sea near the Korean coast and in the Celtic Sea have been recently reported. Study of internal breather dynamics now is one of the topical problems in physical oceanography. The analytical description of long-wave breathers is based on the completely integrable Gardner equation with a positive coefficient of the cubic nonlinearity. We have shown that this coefficient can be indeed positive for the real oceanic stratifications. The long-time breather-like wave evolution within the framework of the Euler equation was numerically simulated in the model of a water tank; the results obtained confirm the predictions of weakly nonlinear theory. The breather generation from the sign-varied perturbation was studied analytically and numerically within the framework of Gardner equation. The breather generation of intermediate length, which occurs in the course of interaction of a soliton of the second mode with the bottom step, was studied within the framework of Euler equation. The propagation and transformation of internal breather-like waves was studied in the idealized, but close to the realistic stratification and bottom profile, taking into account the Earth's rotation. The simulations were performed in parallel both within the framework of weakly nonlinear Gardner equation and within fully nonlinear Euler equation. The breather stability within different models is discussed. The synchronism in the motion of the second mode internal solitary wave and internal breather-like wave of the first mode is also discussed.

Linear and nonlinear equatorial atmospheric waves coupled to moist convection

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Abstract

Equatorial regions in the atmospheres and oceans of rotating planets are special, as they are home to a variety of specific waves trapped in the vicinity of equator. Some of these waves have unidirectional propagation and no dispersion (equatorial Kelvin waves), or unidirectional propagation and weak (strong) dispersion in the long(short)-wave limit (equatorial Rossby waves), others are bi-directional and weakly dispersive in the short-wave limit (inertia-gravity and Yanai waves). Combined with nonlinear effects, this variety of dispersion properties gives rise to a number of archetypal nonlinear wave behaviors for different types of waves: soliton formation, steepening and breaking, resonant interactions, etc, which I will illustrate. Moreover, even the linear problem of wave reflection and transmission over topography becomes highly nontrivial at the equator due to unidirectionality of the main types of long waves, and leads to formation of wave boundary layers, and generation of boundary Kelvin waves.

The main characteristic of the tropical atmosphere of the Earth is its high content in humidity, and pronounced moist convection. The motions due to atmospheric equatorial waves lead to evaporation and condensation of the water vapor, and, thus, are coupled to convection. Such convection-coupled waves are known to strongly influence weather and climate. Within a simple, yet self-consistent and retaining all essential characteristics of the large-scale moist processes model of the tropical atmosphere, the moist-convective rotating shallow water, I will explain how the condensation of water vapor and related heat release modify the dynamics of nonlinear equatorial waves and, inversely, how equatorial waves produce characteristic convection patterns. As an example, I will consider nonlinear dynamics of equatorial waves interacting with topography and Indo-Pacific oceanic warmpool in the vicinity of Maritime Continent, which is known for its crucial influence upon the world's weather and climate.

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